

## Example of a Macroscopical Classical Situation that Violates Bell Inequalities.

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*Summary* - We give an example of a classical macroscopical situation that violates Bell inequalities. The example shows a certain analogy with the system composed of two spin- $\frac{1}{2}$  particles in the singlet spin state.

It was BELL who emphasized that if one wants to make a hidden-variable formalism that reproduces the results predicted by quantum mechanics for two spin- $\frac{1}{2}$  particles in the singlet spin state, then this hidden-variable formalism must be nonlocal<sup>(1)</sup>. This study of Bell was done on the hand of a certain type of inequality which is now called the Bell inequality. It is often thought that this Bell inequality is only violated by systems described by quantum mechanics. We give an example of a classical macroscopical situation that violates Bell inequalities. The example shows a certain analogy with the system composed of two spin- $\frac{1}{2}$  particles in the singlet spin state. This example has been found as a result of a study of separated physical systems<sup>(2)</sup> and<sup>(3)</sup> and supports what is shown in<sup>(2)</sup> and<sup>(3)</sup>, namely that Bell inequalities are satisfied if and only if the two systems under consideration are separated. Whether the systems are quantal or classical is of no importance.

Before giving the example, let us state the definitions necessary to formulate Bell inequalities.

We have a physical system described by a hidden-variable formalism (in the terminology of<sup>(2)</sup> and<sup>(3)</sup>, the system is classical). Let us call  $\Sigma$  the set of states of the system. We will denote yes-no experiments by the greek letters  $\alpha, \beta, \gamma, \delta$ . To every yes-no experiment  $\alpha$  we make correspond a random variable  $X_\alpha$ .

$$X_\alpha: \Sigma \rightarrow \{-1, +1\}$$

such that

$$X_\alpha(p) = +1, \quad \text{if we have the answer « yes »,}$$

$$X_\alpha(p) = -1, \quad \text{if we have the answer « no ».}$$

<sup>(1)</sup> J. S. BELL: *Physics*, **1**, 195 (1965).

<sup>(2)</sup> D. AERTS: *The one and the many*. Towards a unification of the quantum and classical description of one and many physical entities. Doctoral Thesis, Vrije Universiteit Brussel, TENA (1981).

<sup>(3)</sup> D. AERTS: *Description of many physical entities without the paradoxes encountered in quantum mechanics*, submitted to *Found. Phys.* (1981).

If we have two yes-no experiments  $\alpha$  and  $\beta$ , it can happen that the experimental arrangements used to perform  $\alpha$  and  $\beta$  are not incompatible, in this sense it is possible to perform a coincidence experiment on the system. What we do is to perform the experiment  $\alpha$  and  $\beta$  in such a way that the answers of both experiments are obtained at the same time. We define then

$$X_{\alpha\beta}(p) = +1, \quad \text{if we get the answer \{yes, yes\} or \{no, no\} \\ \text{for the coincidence experiment,}$$

$$X_{\alpha\beta}(p) = -1, \quad \text{if we get the answer \{yes, no\} or \{no, yes\} \\ \text{for the coincidence experiment.}$$

In general there will be no connection between  $X_{\alpha\beta}(p)$  and  $X_\alpha(p)$  and  $X_\beta(p)$ , since the coincidence experiment of  $\alpha$  and  $\beta$  is a new experiment.

We suppose now that we have four yes-no experiments,  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$ , such that it is possible to perform coincidence experiments of  $\alpha$  and  $\beta$ , of  $\alpha$  and  $\gamma$ , of  $\delta$  and  $\beta$  and of  $\delta$  and  $\gamma$ . BELL considers the situation where  $\alpha$  and  $\delta$  are experiments done on a system  $S_1$  and  $\beta$  and  $\gamma$  experiments done on a system  $S_2$  such that  $S_1$  and  $S_2$  are localized in different regions of space and such that

$$(1) \quad X_{\alpha\beta}(p) = X_\alpha(p) X_\beta(p),$$

$$(2) \quad X_{\alpha\gamma}(p) = X_\alpha(p) X_\gamma(p),$$

$$(3) \quad X_{\gamma\beta}(p) = X_\gamma(p) X_\beta(p),$$

$$(4) \quad X_{\delta\gamma}(p) = X_\delta(p) X_\gamma(p).$$

He derives then an equality of the following type:

$$(5) \quad |X_{\alpha\beta}(p) - X_{\alpha\gamma}(p)| + |X_{\delta\beta}(p) + X_{\delta\gamma}(p)| \leq 2.$$

What we show in (9) is indeed that requirements (1)-(4) imply that the systems  $S_1$  and  $S_2$  are separated. They are not necessarily satisfied for nonseparated systems.

Let us now give the example of a macroscopical situation that violates Bell inequalities. Let us first define the physical system and the yes-no experiments that we want to consider. We consider a physical system that is a vessel that contains water. We consider the following experiments.

*experiment  $\alpha$* : it consists in testing whether the volume of water contained in the vessel is more than 10 litres. We perform this experiment by emptying the vessel by means of a siphon and collecting the water in a reference vessel of 10 litres. We give the answer « yes » when the water that is flowing from the first vessel to the reference vessel depasses 10 litres, and we give the answer « no » if the water stops flowing before it depasses the 10 litres.

*experiment  $\beta$* : it consists in testing whether the depth of the water in the vessel is more than 15 cm. To perform this experiment we use a tube with a movable piece of wood in the tube. If we put the wood vertically in the vessel till we reach the bottom, the piece of wood will float on the water. When we redraw the tube out of the water, the depth of the water will be indicated by the position of the piece of wood in the tube. We give an answer « yes » if we read a depth of more than 15 cm. We give an answer « no » if this is not the case.

*experiment  $\gamma$* : it consists in testing whether the water is drinkable. We perform this experiment by taking from the vessel a spoonful of water and drink it. We wait five minutes and if we are not ill we give the answer «yes». Otherwise, we give the answer «no».

*experiment  $\delta$* : it consists in testing whether the water is transparent. We again take a spoonful of water, put it in a glass and hold it against a light source. If the light gets through, we give the answer «yes». Otherwise, we give the answer «no».

We shall now consider a physical system consisting of two vessels of water. All the experiments can then be performed on each of these two vessels. We shall show that the system of two vessels shows a certain analogy with the system of two identical spin- $\frac{1}{2}$  particles in the singlet spin state. Of course for the system of the two vessels of water we will from the start show all the hidden variables, so that there will be no mystery at all in what happens. This we cannot do for the spin- $\frac{1}{2}$  particles, because we do not know in which way they are not separated. To have an analogy, we should put the two vessels in a black box and collect only the results of the experiments. Our system consists of two vessels  $V_1$  and  $V_2$  which are of the form of a cube of side 20 cm and which contain water. The two vessels are connected by a tube  $T$  which can contain 16 l of water. The system consisting of  $V_1$ ,  $V_2$  and  $T$  and which is indeed again a vessel is filled with 32 litres of water (16 l in the tube and 8 l in each vessel). The variables that describe the tube can be seen as hidden variables that describe the interaction between  $V_1$  and  $V_2$  (we can for example hide the tube). We immediately see that they are nonlocal hidden variables (they do not belong to  $V_1$  neither to  $V_2$ ). We shall perform experiments  $\alpha$  and  $\delta$  on  $V_1$  and experiments  $\beta$  and  $\gamma$  on  $V_2$  and coincidence experiments of  $\alpha$  and  $\beta$ ,  $\alpha$  and  $\gamma$ ,  $\delta$  and  $\beta$ ,  $\delta$  and  $\gamma$  on the system.

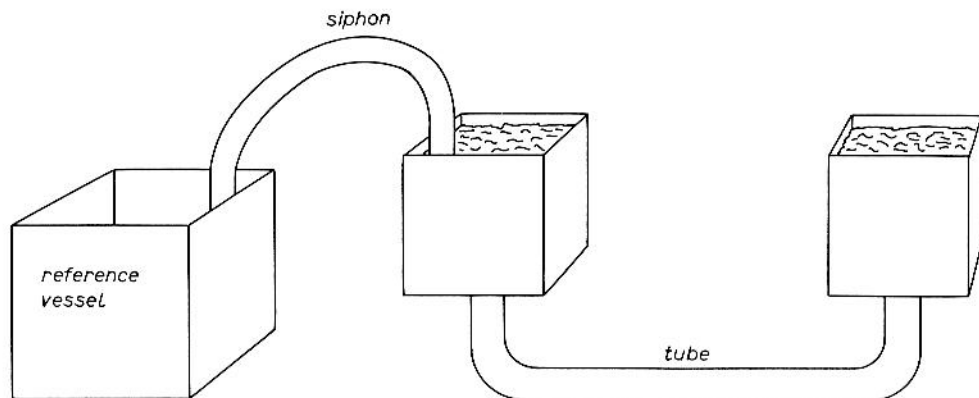


Fig. 1.

As we have already remarked, we say that an experiment is a coincidence experiment if the answers are obtained at the same time.

For the coincidence experiment of  $\alpha$  and  $\beta$  we find

$$X_{\alpha\beta} = -1.$$

Indeed, if we start the experiment, since the water flows from  $V_1$  into the reference vessel, the level of the water in  $V_1$  and  $V_2$  will both go down at the same amount. By the time

that we get 10 l (and this certainly happens since  $V_1$  and  $V_2$  together contain 16 l), there will still be 3 l left in  $V_1$  and 3 l in  $V_2$ . Hence the depth of the water in each vessel will be 7.5 cm. So we find an answer «yes» for  $\alpha$  and an answer «no» for  $\beta$ . (Remark that this whole reasoning is done by using the hidden variables.)

For the coincidence experiment of  $\alpha$  and  $k$  we find

$$X_{\alpha\gamma} = +1.$$

Indeed, if we take a spoonful of water, this will certainly leave more than 10 l in the two vessels.

For the coincidence experiment of  $\delta$  and  $\beta$  we find

$$X_{\delta\beta} = +1.$$

The taking of a spoonful of water will certainly not change the level of the water of less than 5 cm.

For the coincidence experiment of  $\delta$  and  $\gamma$  we find

$$X_{\delta\gamma} = +1.$$

As a consequence

$$|X_{\alpha\beta} - X_{\alpha\gamma}| + |X_{\delta\beta} + X_{\delta\gamma}| = +4,$$

which shows that the Bell inequalities are violated.

The reason is that  $X_{\alpha\beta} \neq X_{\alpha} \cdot X_{\beta}$ .

The reason for this violation is that the two vessels are not separated. The water is in a kind of singlet state, it is not in  $V_1$  neither in  $V_2$ , it is not localized.

We see that if we wanted to give a hidden-variable description (the tube  $T$ , the reference vessel, the siphon), this hidden-variable description would not be local. However, there is no paradox at all, since we know the hidden variables. We easily see that the Einstein-Podolsky-Rosen reasoning is not correct on this system of two vessels. We cannot attribute elements of reality to  $V_1$  by making tests on  $V_2$  and this because the two vessels are not separated. Everybody will agree that these two vessels are not separated. The test  $\alpha$  performed on the vessel  $V_1$  perturbs the state of the water in the vessel  $V_2$ . This perturbation is, however, going on while we make the measurement of the volume. Hence we could not understand this perturbation if we did not understand the mechanism of the measuring process that takes place. For the case of the two spin- $\frac{1}{2}$  particles, we do not have a description of the mechanism of the measuring process. Quantum mechanics just tells us what are the possible states of the system after a measurement.

We also want to mention that also in classical mechanics, we do not have a description of the measuring procedure in general. The vessels  $V_1$  and  $V_2$  are not separated. However, we cannot say that there is an interaction between  $V_1$  and  $V_2$ , in the classical sense of the word «interaction».

The two vessels  $V_1$  and  $V_2$  are localized in different regions of space, as far apart as one wants. The perturbation on the state of the water in  $V_2$  by the test of  $\alpha$  on the vessel  $V_1$  does not depend on the distance between the two vessels, just as in the case of the two particles of spin  $\frac{1}{2}$ .

We can have the image that there is a signal going from vessel  $V_1$  to vessel  $V_2$ . In reality this signal will have a speed equal to the speed of sound waves in water. If

we should, however, consider the water as an incompressible fluid, which is a very good model for the tests  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  that we want to consider, then in this model the speed of the signal will be infinite. The same happens without doubt for the quantum-mechanical description of the two spin- $\frac{1}{2}$  particles in the singlet spin state. The quantum-mechanical model of the measuring procedure is very rough. Often it seems to be understood that the collapse of the wave packet is something happening instantaneously. This is, however, not one of the axioms of quantum mechanics. Quantum mechanics does not teach us anything about the way in which this collapse of the wave packet happens. We are not even at the stage of the perfect fluid in quantum mechanics. Although the two vessels are localized in different regions of space, this is not the case for the water. We can see, however, that certain properties of water are separated. Indeed, the property of being drinkable, the property of being transparent are properties that are separated. One can wonder whether it is not this what happens for spin- $\frac{1}{2}$  particles in the singlet state. Indeed, the only act that is done to separate the two particles in space is making them pass through wave-length filters of a different wave-length. Hence the properties « wave-length » are separated. This does not mean that the two particles are localized in different regions of space, neither that the properties « spin » have to be separated.