THE DESCRIPTION OF SEPARATED SYSTEMS AND QUANTUM MECHANICS AND A POSSIBLE EXPLANATION FOR THE PROBABILITIES OF QUANTUM MECHANICS

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ABSTRACT

It is shown that quantum mechanics cannot provide a mathematical model for the description of separated systems and that this is the origin of the 1935 EPR paradox. We give an explanation for the quantum probabilities by showing that the nonclassical probability calculus of quantum mechanics can be explained as being the consequence of a lack of knowledge about the measurements. We furnish an example of a macroscopic real system that violates Bell inequalities and the Bell locality hypothesis. The system also produces correlations that correspond to events that are spacelike separated.

1. INTRODUCTION

This paper consists of three parts.


We shall show explicitly that quantum mechanics cannot provide a mathematical model for the description of separated systems. It is then demonstrated how this shortcoming of quantum mechanics is at the origin of the 1935 EPR paradox.

1.2. An Explanation for the Probabilities of Quantum Mechanics.

The probabilities of a classical statistical theory are a consequence of our lack of knowledge about the state of the system. Hence they do not conflict with a realistic (even deterministic) description of nature. The probabilities of quantum mechanics cannot be explained in this way (Von Neumann's theorem and later refinements) because they form a non-Kolmogorovian probability model.

We shall show that a lack of knowledge about the measurements gives rise to a non-Kolmogorovian probability model. With this idea of introducing a lack of knowledge about the measurement, we can very easily

construct a macroscopic real system with a quantum probability model (we construct a spin model). It is then argued that the probabilities of any arbitrary quantum system can be explained in this way.

1.3. Bell Inequalities, Separability, Einstein Locality and Local Hidden-Variable Theories.

We shall demonstrate that the violation of Bell inequalities is not a property of quantum systems. Bell inequalities can also be violated by macroscopic classical systems. In fact Bell inequalities can always be violated if one system (classical or quantal) is broken into pieces.

It will also be shown that the production of correlations corresponding to events that are spacelike separated, as in the photon correlation experiments, is not specific for quantum systems. Such correlations can also be produced by classical macroscopic systems.

Finally we shall try to see what is the real meaning of the violation of Bell inequalities and also analyze the Aspect variable-polarizer experiment.

2. THE PROBLEM OF THE DESCRIPTION OF SEPARATED SYSTEMS AND QUANTUM MECHANICS

We employ the concept of separated systems in the following sense: Two systems $s_1$ and $s_2$ are separated if an experiment performed on one of the systems does not change the state of the other system. To be able to use this definition, we first have to introduce the concept of separated experiments. Suppose we have two experiments $e_a$ and $e_b$, and let us denote by $A$ and $B$ the set of their outcomes. In general it is not possible to perform $e_a$ and $e_b$ together. This is so because often the performance of one of the experiments changes the state of the system in such a way that it becomes impossible to perform the other experiment. Sometimes it is however possible to perform $e_a$ and $e_b$ together. This means that there exists an experiment, which we shall denote by $e_{ab}$, with set of outcomes $A \times B$. If we perform the experiment $e_a$ and find an outcome $(x,y) \in A \times B$, then we interpret this as an outcome $x$ for the experiment $e_a$ and an outcome $y$ for the experiment $e_b$.

**Example 1** Consider a system $S$ of two spin-1/2 particles in the singlet spin state. We perform a measurement $e_a$ of the spin of one of the particles in a certain direction and in a certain region of space and a measurement $e_b$ of the spin of the other particle in the same direction but in the opposite region of space. The outcome sets of experiments $e_a$ and $e_b$ are \{yes, no\}, where yes means that the particle has passed the Stern-Gerlach and no that the particle is absorbed (Fig.1). What we have in mind is the well-known experiment proposed by Bohm and since carried out several times to test Bell's inequalities. The experiment $e_{ab}$ consists of performing $e_a$ and $e_b$ simultaneously, and its outcome set is \{yes, yes\}, \{yes, no\}, \{no, yes\}, \{no, no\}. As is shown by experiments and is also predicted by quantum mechanics, for $e_a$ we find always one of the outcomes (yes, no) or (no, yes). For $e_{ab}$, however, we can find the outcome yes and
no both with probability 1/2, and the situation is similar for \( e_b \).

![Diagram of the spin-1/2 particle correlation experiment.](image)

**Fig. 1.** The spin-1/2 particle correlation experiment.

**Example 2** Consider a system \( S \) consisting of two vessels \( V_1 \) and \( V_2 \) that contain each 10 liter of water and are connected by a tube (Fig. 2). The experiment \( e_a \) consists of taking the water out of \( V_1 \) with a siphon and collecting it in a reference vessel \( R_1 \). If we collect more than 10 liter of water, the outcome for \( e_a \) is yes. If we collect less than 10 liter, the outcome for \( e_a \) is no. The experiment \( e_b \) is the same as \( e_a \) but performed on \( V_2 \). The coincidence experiment \( e_a e_b \) consists of performing \( e_a \) and \( e_b \) together. Again it is seen that for \( e_a e_b \) the outcomes are always \((\text{yes}, \text{no})\) or \((\text{no}, \text{yes})\). However, for \( e_a \) one always finds the outcome yes and likewise for \( e_b \).

![Diagram of a correlation experiment with two vessels of water violating Bell inequalities.](image)

**Fig. 2.** A correlation experiment with two vessels of water violating Bell inequalities.

In both examples 1 and 2 we see that some of the combinations of outcomes of \( e_a \) and \( e_b \) are not possible for the experiment \( e_a e_b \). Indeed in both cases yes is a possible outcome for \( e_a \), and yes is a possible outcome for \( e_b \), but (yes, yes) is not possible for \( e_a e_b \). This indicates that the experiments \( e_a \) and \( e_b \) influence each other in a certain sense. When this kind of influence is present, we will say that \( e_a \) and \( e_b \) are non-separated experiments. Hence we can then give the following intuitively clear definition of separated experiments:

**Definition 1** If we have two experiments \( e_a \) and \( e_b \) that can be perfor-
med together and hence there exists an experiment $e_{ab}$, then $e_a$ and $e_b$
are separated for $e_{ab}$ iff:

(i) If $x$ is a possible outcome for $e_a$ and $y$ is a possible outcome
for $e_b$ when the system is in the state $p$, then $(x,y)$ is a possible
outcome for $e_{ab}$ if the system is in the same state $p$ (with state we
mean "pure state").

(ii) If $(x,y)$ is a possible outcome for $e_a$ when the system is in
the state $p$, then $x$ is a possible outcome for $e_a$ and $y$ is a possible
outcome for $e_b$ if the system is in the same state $p$.

Clearly, in both examples 1 and 2 the foregoing definition is not satisfied
so that in both examples $e_a$ and $e_b$ are not separated experiments.

Definition 2 Two systems $S_1$ and $S_2$ are separated iff, for all expe-
riments $e_a$ on $S_1$, and for all experiments $e_b$ on $S_2$, we have that $e_a$ and
$e_b$ are separated.

With this definition of separated experiments and separated systems,
we see that the two spin-1/2 particles in the singlet spin state of exam-
ple 1 are not separated systems, just like the two vessels of water con-
ected by a tube of example 2 are not separated systems. I want to remark
immediately that when two systems are separated, this does not mean at all
that there is no interaction between the systems. No, in general there is
always an interaction, and by means of this interaction the dynamical
change of the state of one system is influenced by the dynamical change
of the state of the other system. In classical mechanics, for example,
all two-body situations are situations of separated bodies. Let us show
this. For a classical system in a state $p$ (pure state) a measurement $e$
ever has a determined outcome $x$. Hence definition 2 becomes:

Definition 3 If we have two experiments $e_a$ and $e_b$ that can be per-
formed together on a classical system $S$, then $e_a$ and $e_b$ are separated iff:

(i) If $x$ is the outcome for $e_a$ and $y$ the outcome for $e_b$ when $S$ is in
the state $p$, then $(x,y)$ is the outcome for $e_{ab}$ when $S$ is in the state
$p$.

(ii) If $(x,y)$ is the outcome for $e_a$ when $S$ is in the state $p$, then
$x$ is the outcome for $e_a$ and $y$ is the outcome for $e_b$ if $S$ is in the
state $p$.

If we now consider two material bodies $S_1$ and $S_2$, with any interaction
between them (gravitational or electromagnetic), then clearly definition 3
is satisfied for all experiments $e_a$ on $S_1$ and experiments $e_b$ on $S_2$. This
shows that our definition of separated systems is very weak, and certain-
ly satisfied by all interacting two-body situations in the macroscopic
world. To provide an even better intuitive feeling of what separated sys-
tems are we note the following: One system is separated from the rest of
the universe, but one system is not separated from the measuring appara-
tus during a measurement.

If we make some very general assumptions on the concept of state of
a physical system, then it can be shown that two systems are separated
iff a measurement performed on one of the systems does not change the
state of the other system.

Hence in this case our definition of separated systems reduces to the one used by APR in their 1935 paper (see theorem 1 of [1]). Let us show now that quantum mechanics cannot provide a mathematical model for such separated systems.

Theorem 1 If e_a and e_b are experiments on a physical system S described by quantum mechanics, then there is always a vector in the Hilbert space of S, such that when S is in the state represented by this vector, e_a and e_b are not separated.

Proof see the proof of theorem 3 of [1].

In [2], [3], and [4] we make an extensive study of separated systems in a more general theory than quantum mechanics. This theory is an elaboration of Piron's approach to quantum logic [5]. It is more general than classical mechanics and quantum mechanics, because we can define a set of axioms such that, when these axioms are satisfied the theory reduces to classical mechanics or to quantum mechanics. The mathematical framework used in this theory is that of a complete lattice with an orthogonality relation. The elements of the lattice represent the properties of the physical system. The axioms to reduce the theory to quantum mechanics are formulated on this lattice of properties. The first two axioms introduce the mathematical structure of an orthocomplementation on the lattice. The third axiom makes the lattice atomic. The fourth axiom makes the lattice weakly modular and the fifth axiom is equivalent to the covering law (or makes the lattice semi modular). What we found in [2], [3], and [4] is that axiom 4 and axiom 5 are never satisfied for the description of separated systems. Let us repeat the main theorem of [2], [3], and [4].

Theorem 2 Suppose that S is a physical system composed of two separated physical systems S_1 and S_2. If the lattice of properties of S satisfies axiom 4 (is weakly modular) or, if the lattice of properties of S satisfies axiom 5 (the covering law), then S_1 or S_2 is a classical system for which the lattice of properties is a Boolean lattice.

Proof see proof of theorem 30 of [2].

The lattice of properties of any system described by quantum mechanics always satisfies both axiom 4 and 5, but is never a Boolean lattice. So, from this theorem it follows that, if S_1 and S_2 are both described by quantum mechanics, then S can never be described by quantum mechanics. This failure of quantum mechanics to describe separated systems is really due to the mathematical structure of quantum mechanics and does not depend on the interpretation. The covering law (axiom 5) is the axiom that introduces the vectorspace structure for the set of states. Hence the set of states of a system composed of two separated systems will not have a vectorspace structure. As a consequence, the superposition principle will not be valid. The theory used in [2], [3], and [4] allows the description of separated systems, since none of the axioms have to be satisfied in this theory.
Let us show now how this shortcoming of quantum mechanics, of not being able to provide a model for separated systems, is at the origin of the 1935 EPR paradox.

To do this we first reanalyze briefly the EPR reasoning in [6]. EPR consider the situation of one system $S$ consisting of two systems $S_1$ and $S_2$, and they use quantum mechanics to describe $S$. They show that, after a measurement $A_2$ on $S_2$, a certain observable $A_1$ of $S_1$ has a sure outcome. And after a measurement $B_2$ on $S_2$ another observable $B_1$ of $S_1$ has a sure outcome. They show that one can construct $A_1$ and $B_1$ in such a way that $[A_1, B_1] \neq 0$. If one now supposes that $S_1$ and $S_2$ are separated systems (separated in our sense, see definition 1), then $A_1$ and $B_1$ had already sure outcomes before the measurements $A_2$ and $B_2$. Since $[A_1, B_1] \neq 0$, this situation (this state) of system $S_1$ cannot be described by quantum mechanics. So there is something wrong, either with the quantum mechanical description of $S_1$ or with the quantum mechanical description of $S$. People who tried to solve the problem by introducing hidden variables thought that something was wrong with the description of $S_1$, and, since $S_1$ is an arbitrary system, then something must be wrong with quantum mechanics in general (referred to as incompleteness by EPR). As we have shown, it is the quantum mechanical description of $S$ which is wrong, in other words, the state of $S$ considered by EPR is not a state of separated systems. Hence we can have two situations.

First situation. The two systems are separated. Then quantum mechanics does not deliver a model for this situation. Correcting quantum mechanics does not happen by adding states to the subsystems (the idea of incompleteness), but by taking states away from the compound system. For the remaining states (all product states) no EPR paradox can be found. This act of retaining only the product states does however not save quantum mechanics. There is an incompleteness in the description of $S$. No states are lacking, but observables are lacking (see [1]).

Second situation. The two systems are not separated. In this case, as we already explained, it is not possible to hold the EPR reasoning. Indeed if we try to attribute an element of reality to one of the systems $S_1$ by making a measurement on the other system $S_2$, then, because the measurement on system $S_2$ can change the state of $S_1$, the element of reality of $S_1$ can be created by the measurement on $S_2$. Hence we cannot continue the EPR reasoning and say that the element of reality was already there before we made the measurement on $S_2$. This step is however necessary if we want to show that $S$ has two elements of reality corresponding to noncompatible observables.

3. AN EXPLANATION FOR THE PROBABILITIES OF QUANTUM MECHANICS

The probabilities of classical statistical theories are a consequence of the lack of knowledge about the state of the system. Can the quantum probabilities also be explained in this way?

This problem has been studied by several people and is usually referred to as the "hidden-variable problem". A lot of no-go theorems have
been proposed and criticized. Most of the no-go theorems are founded on the following fact.

A "lack of knowledge about the state" theory (or a hidden variable theory) gives always rise to
(1) a commutative algebra of observables,
(2) a Boolean lattice of properties,
(3) a Kolmogorovian probability model.

While in quantum mechanics we have a noncommutative algebra of observables, a non-Boolean lattice of properties and a non-Kolmogorovian probability model.

This shows, in my opinion, in a very convincing way that quantum mechanics is not a "lack of knowledge about the state" theory.

The question that we want to raise is the following: Can the quantum probabilities be the consequence of a lack of knowledge about something other than the state of the system? We shall show that this is indeed possible. What we will show more explicitly is that a lack of knowledge about the measurements gives rise to a non-Kolmogorovian probability model. Due to lack of space, we will not be able to prove everything, but a much more detailed study can be found in [7] and [8]. We shall show that the quantum probabilities can be explained in this way. We will give an example of a macroscopic classical system with a lack of knowledge about the measurements having a non-Kolmogorovian probability model. Let us first of all give this macroscopical classical system with a quantum probability model.

We consider a particle with positive charge $q$ that is located on a sphere with radius $r$ at point $(r, \theta, \phi)$. Our physical system is this particle, and the state of the particle is completely determined by the direction $(\theta, \phi)$. Hence there is no lack of knowledge of this state. The measurement $e_{AB}$ consists of the following operation: We choose two particles with negative charges $q_1$ and $q_2$ such that $q_1 + q_2 = Q$. The charge $q_1$ is chosen at random between 0 and $Q$. This represents the lack of knowledge about the measurements. Once the charges $q_1$ and $q_2$ are chosen we put the two particles diametrically on the sphere, such that $q_1$ is at the point $(r, \alpha, \beta)$ and $q_2$ is at the point $(r, \pi-\alpha, \pi-\beta)$.

![Diagram](image)

Fig. 3. The macroscopic classical system of a charged particle on a sphere having a quantum probability model.
Let us call $F_1$ and $F_2$ the coulomb forces of $q_1$ and of $q_2$ on $q$. If the magnitude of $F_1$ is bigger than the magnitude of $F_2$ we assign the outcome $e_1$ to the measurement $e_{\alpha\beta}$. If the magnitude of $F_2$ is bigger than the magnitude of $F_1$, we assign the outcome $e_2$ to the measurement $e_{\alpha\beta}$. The forces $F_1$ and $F_2$ are in the plane through $(r, \theta, \varphi)$, $(r, \alpha, \beta)$, $(r, \pi-\alpha, \pi+\beta)$.

Fig. 4. The plane of the three charges on the sphere.

Let us call $\gamma$ the angle between $(r, \theta, \varphi)$ and $(r, \alpha, \beta)$. Then

$$F_1 = \frac{q_1 \cdot q}{4\pi \varepsilon_0 r^2 \sin^2 \gamma/2} \quad \text{and} \quad F_2 = \frac{q_2 \cdot q}{4\pi \varepsilon_0 r^2 \cos^2 \gamma/2}.$$  

Let us now calculate the probability to get the outcome $e_1$ for $e_{\alpha\beta}$ if the particle $q$ is in the state $(\theta, \varphi)$:

$$P \left( F_1 > F_2 \right) = P \left( \frac{q_1 \cdot q}{4\pi \varepsilon_0 r^2 \sin^2 \gamma/2} > \frac{q_2 \cdot q}{4\pi \varepsilon_0 r^2 \cos^2 \gamma/2} \right)$$

$$= P \left( q_1 \cos \gamma/2 > (q-q_1) \sin \gamma/2 \right)$$

$$= P \left( q_1 \cos \gamma/2 > \frac{Q - q \sin \gamma/2}{Q} \right) \cos \gamma/2.$$  

This is exactly the probability that we would find $e_{\alpha\beta}$ to represent the measurement of the spin of a spin-1/2 particle in the $(\alpha, \beta)$ direction while the particle has spin in the $(\theta, \varphi)$ direction. So we can describe this macroscopical system by a two-dimensional complex Hilbert space. We then represent the state of the particle $q$ in the $(\theta, \varphi)$ direction by means of the vector

$$x_{\theta, \varphi} = \begin{pmatrix} e^{-i\varphi/2} & 1 \varphi/2 \\ e^{-i\theta/2} & e^{\sin \theta/2} \end{pmatrix}$$

and the measurement $e_{\alpha\beta}$ by means of the self-adjoint operator

$$S_{\alpha\beta} = \frac{1}{2} \begin{pmatrix} \cos \alpha & e^{-i\beta} \sin \alpha \\ e^{i\beta} \sin \alpha & -\cos \alpha \end{pmatrix}$$
We can then apply the calculus of quantum mechanics to the description of this macroscopic system. Let us remark again that the state of the particle \( q \) is a pure state and the probability only comes from a lack of knowledge on the measurement \( e_{q} \).

It is possible to show explicitly that the probability model of the spin of a spin-1/2 particle is non-Kolmogorovian (see [8], section 3). In [7] and [8] we also show that it is possible to make such a model for any quantum system (see [8], section 5). This shows that the quantum probabilities can be explained as being the consequence of a lack of knowledge about the measurement.

4. BELL INEQUALITIES, SEPARABILITY, EINSTEIN LOCALITY AND LOCAL HIDDEN-VARIABLE THEORIES.

In [9] we gave an example of a macroscopic system that violates Bell inequalities. Meanwhile we improved the example in such a way that it not only violates Bell inequalities but also produces correlations that correspond to events that are space-like separated. Exactly as the much discussed spin correlations of the EPR experiments on particles in a singlet spin state. The improved example can be found in [1] [10] and [11]. Let us give our example. We consider the system described in the example 2 of section 2. So the vessels of water connected by a tube and the experiments \( e_{a} \) and \( e_{b} \).

![Diagram](image-url)

**Fig. 5.** The system of two vessels of water connected by a tube that violates Bell inequalities.

The coincidence experiment \( e_{ab} \) creates correlations. Indeed, if we find more than 10 liter in \( R_{1} \), then we will find less than 10 liter in \( R_{2} \), and vice versa. The correlations are detected at both sides when the water stops flowing. And the water stops flowing at both sides simultaneously (we put the siphons in the same configuration and we put the vessels horizontal). Hence the events that correspond to the detection of the correlations are space-like separated events. To calculate Bell inequalities we have to introduce two other experiments. The experiment \( e_{a}' \) consists of taking 1 liter of water out of \( V_{1} \) and checking whether the water is transparent. If the water is transparent, then the outcome of \( e_{a}' \) is yes. If it is not transparent, the outcome of \( e_{a}' \) is no. The experiment \( e_{b}' \) is the same as \( e_{a}' \), but performed on \( V_{2} \). We perform then
coincidence experiments $e^{ab}, e^{a'b'}, e^{ab'}, e^{a'b}$. We define the following
random variables: $E = 1$ if $e^{ab}$ gives yes and $E = -1$ if $e^{ab}$ gives no. In
the same way we define $E^{a'}, E^{b'},$ and $E^{a'b'}$. We also define the random
variables for the coincidence experiments: $E^{ab} = +1$ if $e^{ab}$ gives (yes, yes)
or (no, no), and $E^{ab} = -1$ if $e^{ab}$ gives (yes, no) or (no, yes). If the two
vessels of water are in such a state that they contain both 10 liter of
transparant water, then $E^{ab} = -1, E^{a'b} = +1, E^{ab'} = +1,$ and $E^{a'b'} = +1.$
Hence
\[|E^{ab} - E^{a'b'}| + |E^{ab'} + E^{a'b}| = 4 > 2.\]
This shows that Bell inequalities are violated.
Bell inequalities are derived from a locality hypothesis formulated
by Bell in [12]. Let us show that also this Bell locality hypothesis is
violated by our example. This locality hypothesis was formulated for a
deterministic hidden-variable theory. A deterministic hidden-variable
theory is a theory that postulates the existence of states of the system
such that all observables have determined outcomes when the state is
known. Let us denote by $\Lambda$ the set of these states $\lambda$. Hence in such a
theory, $E^{ab}$ has a determined outcome $E^{ab}(\lambda)$ for every state $\lambda$. Bell introduces then the following hypothesis: For all $a, b, c, b$ and all $\lambda$
\[E^{ab}(\lambda) = E^{a}(\lambda)E^{b}(\lambda).\]
This hypothesis means that the result of the experiment $e$ only depends
on the state $\lambda$ of the system and not on the experiment $e^{ab}$. Since this
Bell locality hypothesis implies Bell inequalities, our example must also
violates the Bell locality hypothesis. Let us try to see why this is so.
It is very easy to specify the deterministic hidden-variables for our
system. Indeed, if we specify the diameters $\lambda_1$ and $\lambda_2$ of the two siphons
the outcomes of all the experiments are determined. Hence one can write:
\[E^{ab}(\lambda_1, \lambda_2) = E^{a}(\lambda_1, \lambda_2)E^{b}(\lambda_1, \lambda_2),\]
\[E^{a}(\lambda_1, \lambda_2) = +1 \text{ and } E^{b}(\lambda_1, \lambda_2) = -1 \text{ if } \lambda_1 > \lambda_2,\]
\[E^{a}(\lambda_1, \lambda_2) = -1 \text{ and } E^{b}(\lambda_1, \lambda_2) = +1 \text{ if } \lambda_1 < \lambda_2.\]
This is a correct factorization if one performs the coincidence experi-
ment $e^{ab}$. If one wants to use the same $E^{a}(\lambda_1, \lambda_2)$ to factorize the random
variable $e^{ab}$ from the experiment $e^{ab}$, it does not work anymore. Indeed, $e^{a}$ performed together with $e^{a}$ always gives the outcome yes. This means
that $E^{ab} not only depends on $\lambda_1$ and $\lambda_2$ but also on the fact that we per-
form $e^{a'}$ or $e^{b'}$, and
\[E^{a}(\lambda_1, \lambda_2, b') \neq E^{a}(\lambda_1, \lambda_2, b')\]
since $E^{(\lambda_1, \lambda_2, b)} = +1$ if $\lambda_1 > \lambda_2$ and $E^{(\lambda_1, \lambda_2, b)} = -1$ if $\lambda_1 < \lambda_2$, while
$E^{(\lambda_1, \lambda_2, b') = +1}$ for all $\lambda_1, \lambda_2$. This shows that the Bell locality
hypothesis is not satisfied in our example.
Bell put forward this locality hypothesis having in mind the system
consisting of two spin-1/2 particles in the singlet spin state, thus the
example 1 of section 2. Why do people find this locality hypothesis natural for this system? Because they imagine this system to be a system consisting of two spin-1/2 particles localized in different regions of space. In our opinion this is a wrong image of two spin-1/2 particles in a singlet spin state. Such a system is not a system of one particle with spin localized in one region of space and another particle with spin localized in a separated region of space. This can be seen very easily from the fact that the singlet state is not an eigenstate of some projector $P_a \& P_b$. Hence the singlet state is a much more complicated state.

Often people talk of the principle of separability. One can define the principle of separability in the following way.

Definition 5. The principle of separability is satisfied iff two systems that are localized in separated regions of space are separated (or independent) systems.

Clearly the example of the two vessels of water connected by a tube is not a test for the principle of separability, because the water is not a system consisting of two systems localized in separated regions of space. For the same reason we claim that the photon correlation experiments are not a test for the principle of separability, since two photons in a singlet spin state is not a system consisting of two photons with their spins localized in separated regions of space.

Bell inequalities are often also derived from the principle of Einstein locality (no signal goes faster than light). In this derivation, again a hidden assumption is made. In the derivation of Bell inequalities from the principle of Einstein locality one supposes that the same hidden variables describe the four coincidence experiments $e_{ab}, e_{ab'}, e_{a'b}, e_{a'b'}$. If one again considers the example of the vessels connected by a tube, then one can see very easily that the hidden variables are different for the experiment $e_{ab}$ with hidden variables $\lambda_1, \lambda_2$ than for the experiments $e_{ab'}, e_{a'b}, e_{a'b'}$ (no hidden variables). Hence the hypothesis that the four experiments can be described by the same hidden variables is not justified from any physical principle.

It is of course possible to consider four sets of hidden variables, one set for each experiment, and then construct the Cartesian product of these four sets. In this way one can describe all four experiments. However something else goes wrong in the derivation of Bell inequalities. The probability distribution $p(\lambda)$ of the hidden variables will depend on the experiments. Again Bell inequalities cannot be derived.

We would now like to find the physical reason for the violation of Bell inequalities. Let us look again our example of the vessels of water connected by a tube. We can already see something if we consider the nature of the hidden variables $\lambda_1$ and $\lambda_2$. These are not hidden variables of the state of the water before the experiment, because the state of the water is completely determined by the fact that the volume is 20 liter. And $\lambda_1(\lambda_2)$ is a hidden variable of the experiment $e_1(\lambda_2)$ but not of the experiment $e_{ab'}(\lambda_2)$. This is so because the correlations that are detected by the experiment $e_{ab'}$ were not present before but are created during the experiment. These are the correlations that can violate the Bell locality hypothesis. We propose to call correlations that were not present before the experiment and are created by and during the experi-
ment correlations of the second kind. Correlations that were already
present before the experiment and are only detected by the experiment we
will call correlations of the first kind. Let us give an example of such
correlations of the first kind. Consider a system consisting of two mate-
rial point particles moving in space and having total momentum zero. A
coincidence measurement of the momenta of the individual particles gives
us correlated results. These correlations were however already present
before the coincidence experiment. The experiment only detects the cor-
relations and does not create them. These kind of correlations cannot be
used to violate the Bell inequalities, because the result of an experi-
ment on one of the particles will never depend on what experiment is be-
ing performed on the other particle. Let us summarize this.

If we consider correlations that are created by and during the coin-
cidence experiment \( e \) (correlations of the second kind), then it is pos-
sible to violate Bell inequalities and the Bell locality hypothesis by
means of this coincidence experiment and some other experiment, beca-
use the outcome of experiment \( e \) will in general depend on whether we perform
\( e \) together with \( e \) or with some other experiment \( e' \). If we consider
correlations that were already present before the coincidence experiment
then the Bell locality hypothesis will be satisfied and Bell inequalities
cannot be violated. Of course, in our example Einstein locality is not
violated, because the vessels of water do not allow to transmit a signal.

Let us now show that also the correlations coming from coincidence
spin measurements of two particles in the singlet spin state are created
during the coincidence experiment and hence are correlations of the se-
cond kind. Therefore we carefully analyze the physical meaning of the
singlet spin state of the two particles. Suppose that \( \psi_{\theta \phi} \) and \( \psi_{\phi \theta} \)
represent the spin states of particle 1 and 2 in the \((\theta, \phi)\) and \((\phi, \theta)\) direction,
usually the singlet spin state is written as follows

\[
\psi_s = \frac{1}{\sqrt{2}} [\psi_{(u)p}, \psi_{(d)0}] - \psi_{(d)0}, \psi_{(u)p}]
\]

where \( \psi_{(u)p} = \psi_{0,0} \) and \( \psi_{(d)0} = \psi_{1,0} \). But mathematically this singlet
state can as well be written as follows:

\[
\psi_s = k [\psi_{\theta \phi}^1 \psi_{\phi \theta}^2 - \psi_{\phi \theta}^1 \psi_{\theta \phi}^2] \quad \text{for} \quad \psi_{\phi \theta} \neq \psi_{\theta \phi} \quad \text{and some} \quad k > 0,
\]

\( \psi_s \) does not depend on \((\theta, \phi)\) or \((\phi, \theta)\). This shows that \( \psi_s \) is not a state of
two particles which already have their spin, although it is mathema-
tically constructed by means of \( \psi_{\theta \phi} \) and \( \psi_{\phi \theta} \). \( \psi_s \) is a state of two particles
which do not yet have spin. And the spin is being created by the coin-
cidence experiment. It remains of course very amazing that this spin
creation can be performed experimentally on a macroscopic scale. If we
agree with this image, it is not at all mysterious that the Bell inequal-
ities are violated by the spin experiments.

In both cases, the vessels of water and the particles in the singlet
state, the experiment that creates the correlations divides one system
into two systems. It is possible to show that from the principle of se-
parability follows that experiments performed on two separated systems
never violate Bell inequalities, while for nonseparated systems it is
always possible to construct experiments that violate the inequalities.
This is shown in [1], [2] and [10] (more specifically, see theorem 2 of [1]). Knowing this result, we see that it is very easy to invent other systems that violate the inequalities. We just have to break one system into pieces. It is even possible to construct a macroscopic system that gives exactly the same numerical violation as the quantum system of the particles in the singlet state. This example is presented in [13].

Let us try to summarize. Suppose that we have a system $S$ consisting of two systems $S_1$ and $S_2$. If $S_1$ and $S_2$ are separated, then Bell inequalities and the Bell locality hypothesis are satisfied because coincidence experiments can only detect correlations that were already present before the experiment (correlations of the first kind). If $S_1$ and $S_2$ are not separated, then Bell inequalities and the Bell locality hypothesis can be violated, because we can make coincidence experiments that create correlations (correlations of the second kind).

Einstein locality is not violated by our example of the vessels of water, which proves that experiments showing that quantum mechanics violates Bell inequalities have no bearing whatsoever on the question of violation or nonviolation of Einstein locality. This is certainly so for all the EPR experiments except for the Aspect experiment with varying polarizers. In our opinion, this experiment is a completely different experiment from the others, and therefore we should like to analyze it in the next section.

5. THE ASPECT EXPERIMENT WITH THE VARYING POLARIZERS

We shall show that this Aspect experiment is not a test for quantum mechanics versus an eventual hidden variable theory but is really a test for a new physical property of the system consisting of two photons in the singlet spin state. So we consider the system consisting of two photons in the singlet state and the measurement $e_{ab}$, which is the coincidence experiment with one polarizer in the direction $a$ and one polarizer in the direction $b$. To be able to formulate exactly what Aspect has found, we introduce the following two hypothesis.

Hypothesis A. The act of measurement of the experiment $e_{ab}$ takes more time than the time needed for light to cross the measurement region (this for all $a, b$).

Hypothesis B. The act of measurement of the experiment $e_{ab}$ takes less time than the time needed for light to cross the measurement region.

In our opinion Aspect has shown that hypothesis A is false for the photons in the singlet spin state. Indeed, if hypothesis A were correct, then the coincidence experiment with varying polarizers would not violate Bell inequalities independent of whether the system is described by quantum mechanics or by a classical (hidden-variable) theory.

Everybody will agree that when the system is described by a classical theory, Bell inequalities are not violated. Let us show now that, under hypothesis A, also the quantum mechanical description of the Aspect experiment would not violate Bell inequalities.
If hypothesis A is accepted, then the Aspect experiment with the varying polarizers is one experiment performed on one system (the two particles in the singlet state) with 16 possible outcomes. Indeed, there are four outgoing channels, and in each of the channels we can have a yes and a no outcome. Such an experiment (because it is one experiment with 16 possible outcomes) can always be represented by one self-adjoint operator with 16 different eigenvalues. Let us call this self-adjoint operator D. It is now possible to show that for one experiment represented by one self-adjoint operator D, Bell inequalities are never violated.

Why are Bell inequalities violated by quantum mechanics if we do not accept hypothesis A? Well, suppose that the varying polarizers on the left are a and a' and the varying polarizers on the right are b and b', then, if we do not accept hypothesis A, we have four different experiments e_{ab}, e_{a'b'}, e_{ab'}, and not one experiment with 16 possible outcomes. Indeed, when the flipping between e_{ab} and e_{a'b'} takes less time than the time needed for light to cross the measurement region, we can consider that two experiments e_{ab} and e_{a'b'} have been performed, one after the other, on different systems. In this case quantum mechanics predicts a violation of Bell inequalities because the four experiments being experiments on different systems, are not represented by one self-adjoint operator.

Let us now prove that the experiments described in quantum mechanics by one self-adjoint operator do not violate Bell inequalities. The proof follows immediately from a result that is well known at this moment. The result is the following: if, for any three of the four random variables \( X_{ab}, X_{a'b'}, X_{ab'}, X_{a'b} \), the joint probabilities do exist, then the expectation values of these random variables satisfy Bell inequalities (for a proof of this result see, for example, [15]).

Suppose now that we have one experiment with 16 possible outcomes described by quantum mechanics by means of one self-adjoint operator D. Then it is well known that there exists one classical (Kolmogorovian) probability space, with sample space \( \mathcal{M} \), the spectrum of the self-adjoint operator, and probability measure

\[
m_\mathcal{M} : \mathcal{B}(\mathcal{M}) \rightarrow [0,1], \quad m_\mathcal{M}(E) = \langle \mathbf{F}_E, \mathbf{\gamma} \rangle,
\]

where \( \mathbf{F}_E \) is the spectral measure corresponding to the event \( E \), and \( \mathbf{\gamma} \) is the state of the system (in our case \( \mathbf{\gamma} \) is the singlet spin state).

The four random variables describing the experiments \( e_{ab}, e_{a'b}, e_{ab'}, e_{a'b'} \) can all be described in this one probability space. As a consequence, they all have joint probabilities, and so Bell inequalities are satisfied.

Hence Aspect's experiment with the varying polarizers shows that hypothesis A is not correct, independent of whether we would use quantum mechanics or a hidden-variable theory for the description of the physical system. So there do exist in nature experiments creating correlations more quickly than would be possible with light signals to create the same correlations. These experiments can however not be used to send signals, because they create the correlations, and hence these correlations have no causal connection. To see what I mean, think again of the example of the two vessels of water violating Bell inequalities. The experiment \( e_{ab} \) used in this example also creates correlations that are not causally connected. It is indeed easy to see that also with these
vessels it is not possible to transmit a signal. But in this example the
time needed for a measurement is clearly more than the time needed for
light to cross the measurement region. Hence for this example of the
vessels of water hypothesis A is satisfied. As a consequence this example
can not be used to construct an analogy for the Aspect experiment with
the varying polarizers. Or in other words, if we would construct with
the vessels of water example an analogy for the Aspect experiment (it
means change between experiment $e_a$ and $e_a'$ and between experiments $e_b$ and
$e_b'$, more quickly than the time needed for light to cross the measurement
region), then the analog of the Aspect experiment would be one experiment
with 16 possible outcomes; and hence Bell inequalities would not be vio-
lated.

6. Conclusion

In our opinion there are two quiet different problems that are often
confused, and this confusion is certainly part of the difficulties around
the EPR problem.

One problem is the fact that quantum mechanics is really a new phys-
ical theory introducing a nonclassical (non-Kolmogorovian) probability
calculus. Such a nonclassical probability calculus cannot be found by
means of a hidden-variable theory with hidden variables in the state of
the system. This is the origin of all the no-go theorems for hidden-va-
riable theories. This problem is very old, and was there from the advent
of quantum mechanics (von Neumann's theorem). What we have shown is that,
if one introduces a lack of knowledge about the measurements, then this
gives rise to a nonclassical (non-Kolmogorovian) probability calculus,
even for macroscopic systems. Moreover, the quantum probability model can
be explained in this way for an arbitrary quantum system. The violation
of Bell inequalities is related to this first fact in the following sense.
If we have one system (or two systems considered as one) and if this
system has a classical probability calculus, then, for measurements on
the system, Bell inequalities are always satisfied. So to be able to
violate Bell inequalities by means of a macroscopic system, I had to
construct a macroscopic system with a nonclassical probability calculus.
(in [7] it is shown explicitly that the example of the two vessels of
water which I use in this paper to violate Bell inequalities has a non-
classical probability calculus).

The second problem is the problem of the description of separated
systems. Here we have shown that quantum mechanics cannot deliver a
mathematical model for such separated systems, and this incapacity is at
the origin of the 1935 EPR paradox. So, in our opinion, the original
EPR reasoning does not lead at all to the conclusion that quantum mecha-
nics is incomplete in the sense that there would be hidden variables in
the state of the system. Bell who was convinced by the EPR paper of the
incompleteness of quantum mechanics, in the sense of hidden variables in
the state, started to look for hidden-variable theories of separated
systems. In this way he touched on the first problem, but also on the
second. Indeed, for experiments performed on separated systems, Bell ine-
qualities are always satisfied independent of whether these systems are
classical or quantal. So, if experiments are performed on two systems, and Bell inequalities are violated, we can only conclude that, as a consequence, these systems were not separated before the experiments. For the vessels of water there is no mystery. For the two photons, the experiment being performed with 12 meter between the apparatus, the result is certainly not obvious. But in our opinion the only possible conclusion is that the two photons in the singlet state are not separated. This fact does not however allow us to draw conclusions about general properties (e.g. separability, Einstein locality etc...) of the rest of the world. Indeed there do exist systems that are separated. These systems cannot be described by quantum mechanics.

In our opinion quantum mechanics will really break down in those fields (e.g. chemistry, biology) where separated quantum systems are encountered in profusion.

7. REFERENCES


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