The Guppy Effect as Interference

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Abstract. People use conjunctions and disjunctions of concepts in ways that violate the rules of classical logic, such as the law of compositionality. Specifically, they overextend conjunctions of concepts, a phenomenon referred to as the Guppy Effect. We build on previous efforts to develop a quantum model [?,?,?], that explains the Guppy Effect in terms of interference. Using a well-studied data set with 16 exemplars that exhibit the Guppy Effect, we developed a 17-dimensional complex Hilbert space $\mathcal{H}$ that models the data and demonstrates the relationship between overextension and interference. We view the interference effect as, not a logical fallacy on the conjunction, but a signal that out of the two constituent concepts, a *new* concept has emerged.

Keywords: theory of concepts, quantum cognition, Guppy effect, concept combination, interference

1 The Guppy Effect – Introduction

A concrete formal understanding of how concepts combine is vital to significant progress in many fields including psychology, linguistics, and cognitive science. However, concepts have been resistant to mathematical description because people use conjunctions and disjunctions of concepts in ways that violate the rules of classical logic; i.e., concepts interact in ways that are non-compositional [?]. This is true also with respect to properties (e.g., although people do not rate *talks* as a characteristic property of *Pet* or *Bird*, they rate it as characteristic of *Pet Bird*) and exemplar typicalities (e.g., although people do not rate *Guppy* as a typical *Pet*, nor a typical *Fish*, they rate it as a highly typical *Pet Fish* [?]). This has come to be known as the Pet Fish Problem, and the general phenomenon wherein the typicality of an exemplar for a conjunctively combined concept is greater than that for either of the constituent concepts has come to be called the Guppy Effect, although further investigation revealed that the Pet Fish Problem is not a particularly good example of the Guppy Effect, and that other concept combinations exhibit this effect more strongly [?].

One can refer to the situation wherein people estimate the typicality of an exemplar of the concept combination as more extreme than it is for one of the constituent concepts in a conjunctive combination as *overextension*. One can refer to the situation wherein people estimate the typicality of the exemplar for the concept conjunction as higher than that of *both* constituent concepts as *double overextension*. We posit that overextension is not a violation of the classical logic of conjunction, but that it signals the emergence of a whole new concept. The aim of this paper is to model the Guppy Effect as an interference effect using a mathematical representation in a complex Hilbert space and the formalism of quantum theory to represent states and calculate probabilities. This builds on previous work that shows that Bell Inequalities are violated by concepts [**?**,**?**] and in particular by concept combinations that exhibit the Guppy Effect [**?**,**?**,**?**,**?**,**?**], and add to the investigation of other approaches using interference effects in cognition [**?**,**?**,**?**].

Our approach is best explained with an example. Consider the data in Tab. 1. It is based on data obtained by asking participants to estimate how typical various exemplars are of the concepts *Furniture*, *Household Appliances*, and *Furniture and Household Appliances* [**?**].

		$\mu(A)_k$	$\mu(B)_k$	$\mu(A \text{ and } B)_k$	$\frac{\mu(A)_k+\mu(B)_k}{2}$	θ_k	λ_k	β_k
A=Furniture, B=Household Appliances								
1	*Filing Cabinet*	0.079	0.040	0.062	0.059	87.61	-0.056	-87.61
2	*Clothes Washer*	0.026	0,118	0.078	0.072	84.01	0.055	84.01
3	*Vacuum Cleaner*	0.017	0,118	0.051	0.068	112.21	-0.042	-112.21
4	*Hifi*	0.056	0.079	0.090	0.067	70.58	0.063	70.58
5	*Heated Waterbed*	0.089	0.050	0.082	0.070	79.28	-0.066	-79.28
6	*Sewing Chest*	0.075	0.058	0.061	0.067	94.74	0.066	94.74
7	*Floor Mat*	0.052	0.023	0.031	0.037	100.87	-0.034	-100.87
8	*Coffee Table*	0,100	0.025	0.050	0.062	104.78	0.048	104.78
9	*Piano*	0.084	0.020	0.043	0.052	101.67	0.040	101.67
10	*Rug*	0.056	0.019	0.028	0.037	106.58	0.031	106.58
11	*Painting*	0.057	0.014	0.021	0.035	120.16	-0.024	-120.16
12	*Chair*	0.099	0.030	0.047	0.065	109.41	-0.052	-109.41
13	*Fridge*	0.042	0,117	0.085	0.079	85.23	0.070	85.23
14	*Desk Lamp*	0.066	0.079	0.085	0.072	79.85	-0.071	-79.85
15	*Cooking Stove*	0.037	0,118	0.088	0.078	81.57	-0.066	-81.57
16	*TV*	0.065	0.092	0.099	0.078	61.89	0.075	61.89

Table 1. Interference data for concepts A=*Furniture* and B=*Household Appliances*. The probability of a participant choosing exemplar k as an example of *Furniture* or *Household Appliances* is given by $\mu(A)_k$ or $(\mu(B)_k$, respectively. The probability of a participant choosing a particular exemplar k as an example of *Furniture and Household Appliances* is $\mu(A \text{ and } B)_k$. The classical probability would be $\frac{\mu(A)_k+\mu(B)_k}{2}$. The quantum phase angle θ_k introduces a quantum interference effect. Values are approximated to their third decimal, and angles to their second decimal.

Although Hampton's original data was in the form of typicality estimates, for the quantum model that we built it is more appropriate for data to be in the form of 'good examples'. Thus we calculated from Hampton's typicality data estimates for the following experimental situation. Participants are given the list of exemplars in Tab. 1 and asked to answer the following questions. *Question A* is 'Choose one exemplar that you consider a good example of *Furniture*'. *Question B* is 'Choose one exemplar that you consider a good example of *Household Appliances*'. Finally, *Question A and B* is 'Choose one exemplar that you consider a good example of *Furniture and Household Appliances*'. Hence, concretely, the data in Tab. 1 were not collected by asking the three 'good example'-questions but calculated from Hampton's data, derived from an experiment in which participants were asked to give typicality estimates. This transformation of Hampton's data retains the basic pattern of results because estimated typicality of an exemplar is strongly correlated with the frequency with which it is chosen as a good example [**?**].

2 A Quantum model

In this section we build a quantum model of the Guppy Effect by modeling Hampton's data in complex Hilbert space for the pair of concepts *Furniture* and *Household Appliances*, and their conjunction *Furniture and Household Appliances.* The way in which we calculated the 'good example' data from Hampton's 'typicality' data is by normalizing for each exemplar the typicality estimates of each participant giving rise to an estimate of the extent to which this exemplar constitutes a 'good exemplar'. We then average on all the participants obtaining $\mu(A)_k$, $\mu(B)_k$ and $\mu(A \text{ and } B)_k$ (see Tab. 1). We interpret the resulting values as estimates of the probability that exemplar k is chosen as an answer for *Questions* A, B, and 'A and B', respectively. Tab. 1 gives the probabilities of responses. Hampton's original typicality data, which ranged between -3 and +3, were rescaled to a [0, 6] Likert scale to avoid negative values, and then afterwards normalized and averaged for each of the three concepts (A, B and, 'A and B') (see Tab.1).

The 'good example' measurement has 16 possible outcomes, namely each of the considered exemplars, and hence is represented in quantum theory by means of a self-adjoint operator with spectral decomposition $\{M_k \mid k = 1, \ldots, 16\}$ where each M_k is an orthogonal projection of the Hilbert space $\mathcal{H}$ corresponding to exemplar k from the list in Tab. 1. The concepts *Furniture* and *Household Appliances* are represented by orthogonal unit vectors $|A\rangle$ and $|B\rangle$ of the Hilbert space $\mathcal{H}$, and the combination *Furniture and Household Appliances* is represented by $\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$, which is the normalized superposition of $|A\rangle$ and $|B\rangle$. It is by means of this superposition that the quantum framework can describe how a new concept 'A and B', emerges out of A and B. In the following, the standard rules of quantum mechanics are applied to calculate the probabilities, $\mu(A)_k$, $\mu(B)_k$ and $\mu(A \text{ and } B)_k$

$$\mu(A)_k = \langle A|M_k|A\rangle \quad \mu(B)_k = \langle B|M_k|B\rangle \tag{1}$$

$$\begin{aligned} \mu(A \text{ and } B)_k &= \tfrac{1}{2}(\langle A| + \langle B|)M_k(|A\rangle + |B\rangle) \\ &= \tfrac{1}{2}(\langle A|M_k|A\rangle + \langle B|M_k|B\rangle + \langle A|M_k|B\rangle + \langle B|M_k|A\rangle) \\ &= \tfrac{1}{2}(\mu(A)_k + \mu(B)_k) + \Re\langle A|M_k|B\rangle \end{aligned} \tag{2}$$

where $\Re\langle A|M_k|B\rangle$ is the interference term. Let us introduce $|e_k\rangle$ the unit vector on $M_k|A\rangle$ and $|f_k\rangle$ the unit vector on $M_k|B\rangle$, and put $\langle e_k|f_l\rangle = \delta_{kl}c_k e^{i\gamma_k}$. Then we have $|A\rangle = \sum_{k=1}^{16} a_k e^{i\alpha_k}|e_k\rangle$ and $|B\rangle = \sum_{k=1}^{16} b_k e^{i\beta_k}|f_k\rangle$, and with $\phi_k =$

$\beta_k - \alpha_k + \gamma_k$, this gives

$$\langle A|B\rangle = (\sum_{k=1}^{16} a_k e^{-i\alpha_k}\langle e_k|)(\sum_{l=1}^{16} b_l e^{i\beta_l}|f_l\rangle) = \sum_{k=1}^{16} a_k b_k c_k e^{i(\beta_k-\alpha_k+\gamma_k)}$$
$$= \sum_{k=1}^{16} a_k b_k c_k e^{i\phi_k} \quad (3)$$
$$\mu(A)_k = (\sum_{l=1}^{16} a_l e^{-i\alpha_l}\langle e_l|)(a_k e^{i\alpha_k}|e_k\rangle) = a_k^2 \quad (4)$$
$$\mu(B)_k = (\sum_{l=1}^{16} b_l e^{-i\beta_l}\langle f_l|)(b_k e^{i\beta_k}|f_k\rangle) = b_k^2 \quad (5)$$
$$\langle A|M_k|B\rangle = (\sum_{l=1}^{16} a_l e^{-i\alpha_l}\langle e_l|)M_k|(\sum_{m=1}^{16} b_m e^{i\beta_m}|f_m\rangle)$$
$$= a_k b_k e^{i(\beta_k-\alpha_k)}\langle e_k|f_k\rangle = a_k b_k c_k e^{i\phi_k} \quad (6)$$

which, making use of (**??**), gives

$$\mu(A \text{ and } B)_k = \frac{1}{2}(\mu(A)_k + \mu(B)_k) + c_k\sqrt{\mu(A)_k\mu(B)_k}\cos\phi_k \quad (7)$$

We choose ϕ_k such that

$$\cos\phi_k = \frac{2\mu(A \text{ and } B)_k - \mu(A)_k - \mu(B)_k}{2c_k\sqrt{\mu(A)_k\mu(B)_k}} \quad (8)$$

and hence (**??**) is satisfied. We now have to determine c_k in such a way that $\langle A|B\rangle = 0$. Note that from $\sum_{k=1}^{16}\mu(A \text{ and } B)_k = 1$ and (**??**), and with the choice of $\cos\phi_k$ made in (**??**), it follows that $\sum_{k=1}^{16} c_k\sqrt{\mu(A)_k\mu(B)_k}\cos\phi_k = 0$. Taking into account (**??**), which gives $\langle A|B\rangle = \sum_{k=1}^{16} a_k b_k c_k(\cos\phi_k + i\sin\phi_k)$, and making use of $\sin\phi_k = \pm\sqrt{1-\cos^2\phi_k}$, we have

$$\langle A|B\rangle = 0 \Leftrightarrow \sum_{k=1}^{16} c_k\sqrt{\mu(A)_k\mu(B)_k}(\cos\phi_k + i\sin\phi_k) = 0 \quad (9)$$
$$\Leftrightarrow \sum_{k=1}^{16} c_k\sqrt{\mu(A)_k\mu(B)_k}\sin\phi_k = 0 \quad (10)$$
$$\Leftrightarrow \sum_{k=1}^{16} \pm\sqrt{c_k^2\mu(A)_k\mu(B)_k - (\mu(A \text{ and } B)_k - \tfrac{\mu(A)_k+\mu(B)_k}{2})^2} = 0 \quad (11)$$

We introduce the following quantities

$$\lambda_k = \pm\sqrt{\mu(A)_k\mu(B)_k - \left(\mu(A \text{ and } B)_k - \frac{\mu(A)_k+\mu(B)_k}{2}\right)^2} \quad (12)$$

and choose m the index for which $|\lambda_m|$ is the biggest of the $|\lambda_k|$'s. Then we take $c_k = 1$ for $k \neq m$. We now explain the algorithm used to choose a plus or minus sign for λ_k as defined in (**??**), with the aim of being able to determine c_m such that (**??**) is satisfied.

We start by choosing a plus sign for λ_m. Then we choose a minus sign in (**??**) for the λ_k for which $|\lambda_k|$ is the second biggest; let us call the index of this term m_2. This means that $0 \leq \lambda_m + \lambda_{m_2}$. For the λ_k for which $|\lambda_k|$ is the third biggest – let us call the index of this term m_3 – we choose a minus sign if $0 \leq \lambda_m + \lambda_{m_2} - |\lambda_{m_3}|$, and otherwise we choose a plus sign, and in the present case we have $0 > \lambda_m + \lambda_{m_2} - |\lambda_{m_3}|$. We continue this way of choosing, always considering the next biggest $|\lambda_k|$, and hence arrive at a global choice of signs for all of the λ_k, such that $0 \leq \lambda_m + \sum_{k\neq m} \lambda_k$. Then we determine c_m such that (**??**) is satisfied, or more specifically such that

$$c_m = \sqrt{\frac{(-\sum_{k\neq m} \lambda_k)^2 + (\mu(A \text{ and } B)_m - \frac{\mu(A)_m + \mu(B)_m}{2})^2}{\mu(A)_m \mu(B)_m}} \tag{13}$$

We choose the sign for ϕ_k as defined in (**??**) equal to the sign of λ_k. The result of the specific solution thus constructed is that we can take $M_k(\mathcal{H})$ to be rays of dimension 1 for $k \neq m$, and $M_m(\mathcal{H})$ to be a plane. This means that we can make our solution still more explicit. Indeed, we take $\mathcal{H} = \mathbb{C}^{17}$, the canonical 17-dimensional complex Hilbert space, and make the following choices

$$|A\rangle = \left(\sqrt{\mu(A)_1}, \ldots, \sqrt{\mu(A)_m}, \ldots, \sqrt{\mu(A)_{16}}, 0\right) \tag{14}$$

$$|B\rangle = \Big(e^{i\beta_1}\sqrt{\mu(B)_1}, \cdots, c_m e^{i\beta_m}\sqrt{\mu(B)_m}, \cdots, e^{i\beta_{16}}\sqrt{\mu(B)_{16}}, \sqrt{\mu(B)_m(1-c_m^2)}\Big) \tag{15}$$

$$\beta_m = \arccos\left(\frac{2\mu(A \text{ and } B)_m - \mu(A)_m - \mu(B)_m}{2c_m\sqrt{\mu(A)_m\mu(B)_m}}\right) \tag{16}$$

$$\beta_k = \pm\arccos\left(\frac{2\mu(A \text{ and } B)_k - \mu(A)_k - \mu(B)_k}{2\sqrt{\mu(A)_k\mu(B)_k}}\right) \tag{17}$$

where the plus or minus sign in (**??**) is chosen following the algorithm introduced for choosing the plus and minus sign for λ_k in (**??**). Let us construct this quantum model for the data in Tab. 1. The exemplar that gives the biggest value of $|\lambda_k|$ is *TV*, and hence we choose a plus sign and get $\lambda_{16} = 0.0745$. The exemplar that gives the second biggest value of λ_k is *Desk Lamp*, and hence we choose a minus sign, and get $\lambda_{14} = -0.0710$. Next comes *Fridge* having $|\lambda_{13}| = 0.0698$, and since $\lambda_{16} + \lambda_{14} < 0$, we choose a plus sign for λ_{13}. We determine in a recursive way the signs for the remaining exemplars. Tab. 1 gives the values of λ_k calculated following this algorithm. From (**??**) it follows that $c_{16} = 0.564$.

Making use of (**??**), (**??**), (**??**) and (**??**), and the values of the angles given in Tab. 1, we put forward the following explicit representation of the vectors $|A\rangle$ and $|B\rangle$ in $\mathbb{C}^{17}$ representing concepts *Furniture* and *Household appliances*.

$$|A\rangle = (0.280, 0.161, 0.131, 0.236, 0.299, 0.274, 0.229, 0.316, 0.289, 0.236, 0.238, 0.315, 0.205, 0.257, 0.193, 0.255, 0) \tag{18}$$

$$|B\rangle = (0.200e^{-i87.61^\circ}, 0.343e^{i84.01^\circ}, 0.343e^{-i112.20^\circ}, 0.281e^{i70.58^\circ}, 0.225e^{-i79.28^\circ}, 0.242e^{i94.73^\circ}, 0.151e^{-i100.87^\circ}, 0.157e^{i104.78^\circ}, 0.140e^{i101.67^\circ}, 0.137e^{i106.58^\circ}, 0.119e^{-i120.16^\circ}, 0.174e^{-i109.41^\circ}, 0.342e^{i85.23^\circ}, 0.280e^{-i79.85^\circ}, 0.344e^{-i81.57^\circ}, 0.171e^{i61.89^\circ}, 0.250). \tag{19}$$

This proves it is possible to make a quantum model of the [**?**] data such that the values of $\mu(A \text{ and } B)_k$ are determined from the values of $\mu(A)_k$ and $\mu(B)_k$ as a consequence of quantum interference effects. For each exemplar k, the value of θ_k in Tab. 1 gives the quantum interference phase.

3 Visualization of Interference Probabilities

A previous paper provided a quantum representation of the concepts *Fruits* and *Vegetables* and their disjunction *Fruits or Vegetables*, and gave a way to graphically represent possible quantum interference patterns that result when concepts combine [?]. Here we follow this procedure to generate a graphical representation for the concepts *Furniture*, *Household Appliances*, and their conjunction *Furniture and Household Appliances*. Each concept is represented by complex valued wave functions of two real variables $\psi_A(x, y)$, $\psi_B(x, y)$ and $\psi_{A\mathrm{and}B}(x, y)$. We choose $\psi_A(x, y)$ and $\psi_B(x, y)$ such that the square of the absolute value of both wave functions is a Gaussian in two dimensions, which is always possible since we only have to fit 16 values, namely those of $|\psi_A|^2$ and $|\psi_B|^2$ for each of the exemplars of Tab. 1. These Gaussians are graphically represented in Figs. 1 a) and 1 b), and the exemplars of Tab. 1 are located in spots such that the Gaussian distributions $|\psi_A(x, y)|^2$ and $|\psi_B(x, y)|^2$ properly model the probabilities $\mu(A)_k$ and $\mu(B)_k$ in Tab. 1 for each of the exemplars. For example, for *Furniture*

Fig. 1. A representation of our quantum model of *Furniture*, *Household Appliance* and *Furniture and Household Appliance* by a double slit interference situation. The brightness of the light source in a region corresponds to the probability that an exemplar in this region is chosen as a 'good example' of the concept *Furniture* in figure a), *Household Appliance* in figure b), and *Furniture and Household Appliance* in figure c). Numbers indicate the exemplars as numbered in Table 1.

(Fig. 1 a)), *Coffee Table* is located in the centre of the Gaussian because it was most frequently chosen in response to *Question A*. *Chair* was the second most frequently chosen, hence it is closest to the top of the Gaussian. Note that in Fig. 1 b) there is one point labelled by X, which is the maximum of the Gaussian representing $\mu(B)$. We preferred not to locate the highest value of typicality by the maximum of the Gaussian, because doing so did not lead to an easy fit of both Gaussians. For *Household Appliances*, represented in Fig. 1 b), X is located in the maximum of the Gaussian, and since *Clothes Washer* and *Vacuum Cleaner* are the most frequently chosen (with exactly the same frequency) they are located closest to X at an equal distance radius. *Cooking Stove* was the third most frequently chosen, then *Fridge* and so on, with *Painting* as the least chosen 'good examples' of *Household Appliances*. Metaphorically, we could regard the graphical representations of Figs. 2 a), 2 b) as the projections of a light source shining through two holes such that a screen captures it and the holes make the intensity follow a Gaussian distribution when projected on the screen. The centre of the first hole, corresponding to *Furniture*, is located where exemplar *Coffee Table* is at point $(0, 0)$, indicated by 8 in both figures. The centre of the second hole, corresponding to *Household Appliances*, is located where point X is at (10,4), indicated by 17 in both figures. In Fig. 1 c) the data for *Furniture and Household Appliances* are graphically represented. This is not 'just' a normalized sum of the two Gaussians of Figs. 2 a) and b), since it is the probability distribution corresponding to $\frac{1}{\sqrt{2}}(\psi_A(x,y)+\psi_B(x,y))$, which is the normalized superposition of the wave functions in Figs. 2 a) and b). The numbers are placed at the locations of the different exemplars, according to the labels of Tab. 1, with respect to the probability distribution $\frac{1}{2}|\psi_A(x,y)+\psi_B(x,y)|^2 = \frac{1}{2}(|\psi_A(x,y)|^2 + |\psi_B(x,y)|^2) + |\psi_A(x,y)\psi_B(x,y)|\cos\theta(x,y)$, where $|\psi_A(x,y)\psi_B(x,y)|\cos\theta(x,y)$ is the interference term and $\theta(x,y)$ the quantum phase difference at (x,y). The values of $\theta(x,y)$ are given in Tab. 1 for the locations of the different exemplars. The interference pattern in Fig. 1 c) is very similar to well-known interference patterns of light passing through an elastic material under stress. In our case, it is the interference pattern corresponding to *Furniture and Household Appliances*. Bearing in mind the analogy with the light source and holes for Figs. 1 a) and b), in Fig. 1 c) we can see the interference pattern produced when both holes are open. (For the mathematical details – the exact form of the wave functions and the calculation of possible interference patterns – and other examples of conceptual interference, see [**?**].)

4 Interpretation of Interference in Cognitive Space

If we consider equations (**??**) and (**??**), which are the fundamental interference equations used in our quantum modeling, we can see that $\mu(A \text{ and } B)$ becomes equal to the average $\frac{1}{2}(\mu(A)+\mu(B))$ in case of no interference, i.e. if the interference terms are zero. This means that the 'no interference situation' does not coincide with the description of the conjunction by means of the minimum rule of fuzzy set theory. Also when we consider the double slit situation, a classical particle passing through the situation with both slits open, gives rise to a probability distribution on the screen which is equal to $\frac{1}{2}(\mu(A)+\mu(B))$, i.e. the average of the probabilities found with one of the two slits open. Hence, in our theoretical interference quantum model, and equally so in its double slit representation, it is 'the average' which plays the role of the 'classical default', and not the 'minimum', what one would expect to be the case if the conjunction would be modeled along fuzzy set theory.

deviations.jpg

Fig. 2. a) Estimations of concept combination probabilities. The horizontal axis corresponds to the exemplar label denoted by k in Tab.1 and the vertical axis measures estimated probability. The two grey curves represent the minimum and maximum for each k of the probabilities $\mu(A)_k$ and $\mu(B)_k$. The black curve represents the probability $\mu(A \text{ and } B)_k$ obtained from the data, and the dashed curve represents the average between $\mu(A)_k$ and $\mu(B)_k$. b) Comparison between the concept conjunction probability, and the classical average and minimum probabilities of the concepts A and B. $\bar{\mu}_k = \frac{\mu(A)_k+\mu(B)_k}{2}$ is the the classical average probability (third column) and $\min_k = \min\{\mu(A)_k, \mu(B)_k\}$ is the minimum probability (fifth column). The fourth (sixth) column shows the deviation of the average (the minimum) with respect to the concept combination probability. The probability $\mu(A \text{ and } B)_k$ deviates 0.011 from the average $\bar{\mu}_k$, but 0.026 from the minimum.

This is an aspect of our model which needs further explanation. We start this explanation by making two remarks. First, also in earlier work when the disjunction was modeled by interference in a similar way than how we model here the conjunction, the same situation arises, again it is the average which is the classical default, and not the maximum which would follow from a fuzzy set theory modeling of the disjunction [**?**]. Second, if we consider Hampton's data, the average $1/2(\mu(A) + \mu(B))$ is effectively closer to the frequency of the combined concept $\mu(A \text{ and } B)$ than the fuzzy set minimum value. More concretely, on average, the probability for the combined concept differs 0.011 from the classical average, but 0.026 from the fuzzy set minimum measure (Fig.-Tab. 2). Also a calculation of the correlation between the probability for the combined concept and the average and the minimum, resulting in 0.899 and 0.795 respectively, indicates the same, namely that experimentally 'the average is a better estimate than the minimum'.

That the average is the classical default in our quantum model, and in the double slit representation of it, and that the average is also a better experimental approximation than the minimum, indicates that the connective 'and' when forming a conjunction of concepts does not play the role that we imagine it to play intuitively and from our experience with logic. The same applies for the connective 'or' when forming a disjunction of concepts, also it does not play the role we imagine it to play from our intuition and our experience with logic [**?**].

A very similar phenomenon was identified for experimental data of Hampton testing for membership weights of exemplars with respect to conjunctive and disjunctive combinations of pairs of concepts. It was resolved by showing that the state space is a Fock space with two sectors, the first sector describing this 'non logical and interference role' of conjunction and disjunction, with indeed the average as classical default, and a second sector describing the logical role of conjunction and disjunction, with minimum and maximum as classical defaults in the case of conjunction and respectively disjunction, and quantum entanglement as quantum effect [**?**]. We believe that also for the approach presented here this is the state of affairs, and that we have only described the 'first sector Fock space' part in the present article, hence the interference part, with the average as classical default, and a role of conjunction that is not the one of logic. The model in the present article, since it describes the interference part in the first sector of Fock space, but not the entanglement part in the second sector of Fock space, can be seen as complementary to an entanglement quantum model that was worked out for the Pet-Fish concept combination in a tensor product Hilbert space [**?**].

An more intuitive way of looking at this situation is that when it comes to first sector of Fock space effects, hence interference effects, participants mainly consider '*Furniture and Household Appliances*' in its root combination '*Furniture–Household Appliances*', without taking into account the 'and' as a logical connective. The 'and' merely introduces an extra context on this root combination, which, for example, will be different from the extra context introduced by the 'or' on the root combination.

At first sight it might seem that our interference quantum model does not allow order effect to appear in the experimental data, while we know that they do appear in real experiments. More concretely, experiments on the combination 'A and B' will often lead to different data than experiments on the combination 'B and A'. These order effect can without problems be modeled in our interference approach, because in this first sector of Fock space, although 'A and B' and 'B and A' will be described by the same superposition state, the phase of this state will be different, leading to different interference angles, and hence different values for the collapse probabilities. This is the way the first sector of Fock space interference model copes in a very natural way with order effects.

The double slit representation that we have built also helps in clarifying aspects of the situation and thus provides original insight into concept combination. The two slits stand respectively for *Furniture* and *Household Appliances* and specific positions on the detection screen where the interference pattern is formed are the measuring locations for the various exemplars. We can easily understand by analyzing the double slit situation that the human mind does not work like the classical particle equivalent. If this were the case, we would be able to see this at the level of the individual answers. Each individual would simply substitute the combined concept *Furniture and Household Appliances* by one of the two constituent concepts – in a manner similar to the classical particle that must pass by one of the two slits. This would precisely result in a perfect average, and hence no interference, as in the case of a classical double slit. However, on many occasions individual judgements of typicality for the conjunction deviate from the average, in a manner similar to how the statistical average of typicality deviates from the average. The interference is therefore already operating at the level of the individual, similar to the interference pattern observed in quantum mechanics even with single quantum 'particles' in a double slit set-up [?]. Physicists introduced the term 'self-interference' to indicate this behavior. The above suggests that the individual ponders each of the constituents of a combined concept, and the notion of self-reference would hence make sense for the human mind combining a pair of concepts.

We finish this reasoning by mentioning that elsewhere, the origin of conceptual interference effects, and their implications for cognition and creativity, have been analyzed and discussed, adding elements to the analysis made in the present paper [?,?,?,?,?].

Let us finish this section by returning to the original pre-occupation with overextension of the conjunction. Since from our analysis follows that the average is a first order and stronger classical default, namely the default of the first sector of Fock space, than the minimum, only being the classical default of the second sector of Fock space, the notion of 'overextension' no longer covers correctly the 'deviation from classicality', but an interesting relation with interference can be found. Overextension takes place when

$$\mu(A \text{ and } B)_k - \min\{\mu(A)_k, \mu(B)_k\} > 0$$

which is equivalent to

$$\max\{\mu(A)_k, \mu(B)_k\} > \frac{\mu(A)_k + \mu(B)_k}{2} - \Re\langle A|M_k|B\rangle$$

When the average modulated by the interference term cannot equal the largest of the constituent typicalities, the overextension occurs. This is consistent with the contention that 'Conjunctions tend to be overextended to include exemplars that are good members of one class, but are marginal to the other' [?]. Only for double overextension with certainty is interference necessary. On the other hand, situations where one of the concepts and the conjunction are estimated to be very atypical, while the other concept is estimated highly typical, a situation traditionally considered unproblematic, could require a large interference deviation from the classical average.

5 Conclusions

We have presented a quantum model that demonstrates how the Guppy Effect can be modeled as interference. A data set for two concepts and their conjunction – with an ontology of 16 exemplars – was modeled in a 17-dimensional Hilbert space $\mathcal{H}$. The non-compositionality of the conjunction of concepts was identified by its close convergence to the classical average of probabilities, while the quantum interference appears as a modulation to fit the effect of the logical connectives. Our core finding is that this effect produces a quantifiable deviation from classical analyses, signalling the emergence of a new concept. One of the implications is that in some situations, particularly when new content emerges, cognitive processes cannot be described using classical logic.

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