Spectral Resolutions in Order Unit Spaces and Generalized Hermitian Algebras

David J. Foulis∗ Sylvia Pulmannová†

We present a generalization and a more algebraic version of the non-commutative spectral theory of Alfsen and Shultz [1]. An order-unit space is called spectral if it is enriched by a compression base with the comparability and projection cover property [2, 3, 4]. We show that each element in a spectral order-unit space determines and is determined by a spectral resolution which is a nonempty closed bounded subset of the real numbers [5].

We introduce the notion of a generalized Hermitian (GH) algebra as a generalization of the real algebra of bounded Hermitian operators on a Hilbert space. Among the examples of GH-algebras are ordered special Jordan algebras, JW-algebras and AJW-algebras, but unlike these more restricted cases, a GH-algebra is not necessarily a Banach space and its lattice of projections is not necessarily complete. Unit intervals of a GH-algebra can be identified as effect algebras, and the projection lattices are σ-complete orthomodular lattices. We show that GH-algebras are spectral order-unit spaces and that they admit a substantial spectral theory. Moreover, we show that maximal commuting subsets of elements of a GH-algebra are related to blocks in its projection lattice and prove a Gelfand-Naimark representation theorem for commutative GH-algebras [6, 7].

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References

∗Emeritus Professor of Mathematics and Statistics, University of Massachusetts, USA; e-mail: foulis@math.umass.edu
†Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovakia; e-mail: pulmann@mat.savba.sk