Mathematics as Liberal Education: Whitehead and the Rhythm of Life

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Abstract: In several of his works, Alfred North Whitehead (1861-1947) presents mathematics as a way of learning about general ideas that increase our understanding of the universe. The danger is that students get bogged down in its technical operations. He argues that mathematics should be an integral part of a new kind of liberal education, incorporating science, the humanities, and “technical education” (making things with one’s hands), thereby integrating “head-work and hand-work.” In order to appreciate the role mathematics plays in modern science, students should understand its diverse history which is capable of bringing abstract ideas to life. Moreover, mathematics can discern the alternating rhythms of repetition and difference in nature constituting the periodicity of life. Since these same rhythms are to be found in his theory of learning as growth, there appears to be a pattern linking Whitehead’s approach to mathematics and his educational philosophy.

KeyWords: Mathematics, liberal education, Whitehead, rhythmic cycles of growth, technical education, integrating the curriculum, history of mathematics, periodicity, rhythm of life.

“… one forgets that the historical fact, as it actually exists and as the historian actually knows it, is always a process in which something is changed into something else. This element of process is the life of history.” (Collingwood, 1961, p.163)

1. Introduction

The idea that mathematics is a central part of education is not new. It has a long history reaching back to such ancient civilizations as China, where mathematics was included in the Confucian education of scholars as administrators in the civil service from the 8th to the 3rd centuries BCE; to Egypt, where advanced mathematics, physics, and astronomy were utilized in transmitting the cultural heritage; to Sumeria (in modern day Iran), where scholars were familiar with the sciences as well as legal and literary studies; to India, whose advanced forms of education included logic and philosophy; and to Greece where the study of higher education, influenced by Socrates and Plato, afforded pride of place to
mathematics, dialectics, and metaphysics (Axelrod, 2002, pp.10, 9, 11). The elites in all of these societies recognized the power of mathematics to unlock the secrets of the universe in different ways: to chart the movement of the stars, provide measurements for building the pyramids or construct temples that observed the principles of the Golden Rectangle. It is also worth pointing out that there were universities at Al-Azhar (in Egypt) and Timbuktu (in modern day Mali), which predated those in Europe and included science and mathematics in the curriculum (van den Berghe, 1973; Al-Azhar University, Cairo, 2012; University of Timbuktu, 2012).

In the 13th century at the mediaeval university of Paris, arithmetic, geometry, astronomy, and music formed the quadrivium, or the meeting place of four roads or four ways. The close relationship among these four arts was integral to the curriculum - a token of respect for the ancient Greek idea that they were worthy of pursuit by “free men” (Freudenthal, 1991; cited in Fyhn, Sara Eira, & Sriraman, 2011, p.190). Together with the trivium (rhetoric, grammar, and logic), these “liberal arts” formed the basis for the study of philosophy and theology, “the queen of mediaeval studies” (Gutek, 1995, p.102; Haskins, 1960, p.27). In the modern world, however, and more particularly since the rise of scientific materialism in the 17th century, mathematics has become separated from those subjects considered to constitute a liberal education, namely the arts and humanities (1). This fragmentation of knowledge has led to what C.P. Snow once called “The Two Cultures,” a divide between the natural sciences, which seek to explain the universe through inductive and deductive reasoning, and the humanities whose goal is an aesthetic appreciation of nature and the literature, art, and cultures of humankind. I think
it fair to say that one of Whitehead’s goals in both his writings on mathematical education and his account of education in general is to try to bridge this gap.

In this paper I show how Whitehead considered mathematics as a way of learning about general ideas that increase an understanding of the universe. In order to prevent students getting bogged down in its technical operations, he argued that it be an integral part of a new kind of liberal education. This rejuvenated education should possess the following characteristics: be useful in terms of the students’ experience; “liberal” in the sense of not being rushed; observant of the history of mathematics, which can bring abstract ideas to life; and provide the platform for synthesizing scientific, literary, and “technical education” (making things with one’s hands), thereby integrating “headwork and handwork.” Like John Dewey (1859-1952) who discerned the rhythms of nature in various forms of art (2005, pp.153-5), Whitehead believed that mathematics could unlock the alternating rhythms of repetition and difference in nature constituting the periodicity of life. Since these same rhythms are to be found in his theory of learning as growth, there appears to be a pattern linking Whitehead’s approach to mathematics and his educational philosophy.

2. Teaching and Learning Mathematics

In the period after leaving the University of Cambridge for London in 1910, Whitehead gave several lectures and wrote An Introduction to Mathematics about the importance of teaching and learning mathematics. The book was first published in 1911 when Russell and Whitehead were still embroiled in the publication of Principia Mathematica. But the goal of An Introduction to Mathematics was quite different; namely, to introduce university students to some of the main ideas of mathematics and their application to the
natural world “disentangled from the technical procedure which has been invented to facilitate their exact presentation in particular circumstances.” As he goes on to explain, it would be “an error to confine attention to technical processes, excluding consideration of general ideas” (1958, pp.1, 2). Students should come to appreciate the importance of general ideas in mathematics both for their intrinsic value and their applications to real life unencumbered by the technical details surrounding them.

Whitehead’s interest in the teaching of mathematics may well have been spurred in part by the need to find a faculty position, since it was not until 1914 that he secured a five year term as Professor of Applied Mathematics at Imperial College (Grattan-Guinness, 2010, p.250), where he remained until he was offered a position in the philosophy department at Harvard in 1924 (Hendley, 2010, p.76). More importantly, however, he saw the need for changes in the ways an elementary course in mathematics was taught at the university level lest it stifle the interest of learners, inhibit their progress, and end in total frustration. At the same time, Whitehead thought that “mathematical ideas, because they are abstract, supply just what is wanted for a scientific description of the course of events” (1958, p.5) in the form of laws of nature, which enable the ongoing development of natural science. He believed these abstract ideas could be taught in ways that make them come alive, but in order to understand how, it is necessary to comprehend the rudiments of his educational thought.

3. Abstract Ideas and the Rhythmic Cycles of Growth

In general terms, Whitehead’s proposals for reforming mathematical pedagogy are consistent with the educational philosophy he was developing during the time he was at the University of London. A familiar theme that runs throughout his work is the need to
relate abstract ideas to the concrete experience and interests of students, so as to avoid their becoming inert. “Inert ideas,” he explains, “are merely received into the mind without being utilized, or tested, or thrown into fresh combinations,” so that instead of engaging students who can then use them in imaginative ways capable of enhancing their experience, they become lifeless and result in “mental dryrot.” Indeed, by the time Whitehead prepared the chapters that became The Aims of Education, a work that includes an essay on the mathematical curriculum and several others on science, he suggested that “the whole book is a protest against dead knowledge, that is to say, against inert ideas” (1957b, pp.1,2,v). Good teaching means that one does not introduce precise ideas too early in the process of learning lest they become inert and fail to capture the students’ imagination. The problem here is to achieve a balance between cultivating the breadth of interest of the student and the demands of discipline-based knowledge because, as he later put it in Process and Reality, “the training which produces skill is so very apt to stifle imaginative zest” (1957a, p.399).

Unless students experience “the joy of discovery” (1957b, p.2) in a cycle of romance, whose distinguishing rhythm is the freedom to inquire on one’s own or with others, they are likely to stop learning because their curiosity will have been squelched. As part of this process of discovery in which adventure is dominant, students should have the freedom to pose questions for themselves, seek answers that enhance their experience, and strengthen their interests in ways that further a sense of wonder. The role of the teacher here is a somewhat muted one, guiding students where necessary but refraining from using a heavy hand that impedes the process of growth (p.32). Only when this cycle has achieved its full course are students likely to appreciate the need to learn the
“grammar” of any discipline like mathematics, its rules and procedures, which comprise the cycle of precision and make further knowledge possible. The distinguishing rhythm of precision is discipline, or more precisely a self-discipline which comes from within the learner. While precision is a necessary phase in the process of learning, it requires pushing forward swiftly lest the love of learning disappear. As the student gains the ability to pursue knowledge in a disciplined manner, s/he moves to the third of the overlapping cycles and learns to relate abstract principles and ideas (of mathematics, say) to concrete circumstances, including her own experience. Once again, the rhythmic pulse of this cycle is freedom but a broader, deeper freedom than in romance, furnished by the knowledge and experience gained in the previous cycles (Woodhouse, 1999, p.193). Nor is the cyclical process finished, since the student in generalization is once again approaching a romantic understanding of the subject matter and is capable of pursuing a lifelong progress through the cycles.(2) I return to a discussion of the cycles of growth and their distinctive rhythm when discussing the periodicity of nature later in the paper.

4. A Rhythm to Mathematics Teaching?

What, then, can we take from Whitehead’s account of learning with regard to teaching mathematics? First, his emphasis on the need to begin the subject in ways emancipated from confusing “technical procedure[s]” is consistent with his view of romance as the initial cycle of learning. Where students are not given the time to appreciate some of the joys of learning mathematics for themselves, they are likely to become dispirited. Experience from everyday life enables them to appreciate otherwise complex concepts like the graphical representation of vectors by straight lines by considering a moving boat with a man walking across its deck. Similarly, Whitehead (1958) uses a train to illustrate
continuous and discontinuous functions in its journey from London when passing a
certain number of stations in a certain amount of time (an example which was still used
when I was taught mathematics at school) (pp.37, 111-13). In each case, students learn
about the abstractions of mathematics by means of concrete examples from which they
draw meaning as they explore the ideas further. The study of mathematics can then come
alive as a sense of joy and discovery permeates their work, providing a strong
appreciation for its importance and application to life.

Once students become familiar with concrete examples of the general ideas of
mathematics by freely utilizing them for themselves, they may begin to see the need for
greater precision in their work. In a lecture given in 1912, titled “The Place of
Elementary Mathematics in a Liberal Education,” Whitehead suggested that students
should learn how logic and mathematics apply to life by means of what he calls “a
precise perception of the nature of the abstractions acquired by constant use of them,
illumined by explanations and finally by precise statements” (1965, p.35). The precision
required for the constant use of differential calculus, for example, requires explanations
of its abstractions by teachers, a keen understanding on the part of students, and the kind
of self-discipline which distinguishes this cycle of growth and enables further learning.
Precise understanding of the calculus or any other of the general ideas of mathematics
may need considerable self-discipline, but it should not slow students down unduly lest it
become a burden that prevents a broader understanding of their significance.

As they move from the cycle of precision to generalization, students learn both
the interconnection of ideas by utilizing “trains of reasoning” and their application “to the
course of nature conceived in its widest sense as including human society.” The goal of
this process is to generate what Whitehead calls “a capacity to apply ideas to the concrete
universe … [so that] half of the teaching of modern history [for example] should be
handed over to the mathematicians” and reduced to the study of statistics and graphs.
The use of such techniques illuminates the quantitative factors in the social forces
shaping the industrialized world - from trade to crime, harvests to health, and population
to prices (1965, p.35). Generalization of this kind may well provide the kind of “breath
of reality” to the study of mathematics, which Whitehead is advocating, but it could be
problematical had he not left room for the qualitative study of history. Among those who
proposed such an approach was R.G. Collingwood (1889-1943), a contemporary of
Whitehead’s. Collingwood (1961) saw the task of the historian as one of penetrating the
thoughts and emotions of historical figures by means of an imaginative process that re-
enacts their “inner” reflections. The complex events which emerge as objects of study
also have an “outside” to them – actions like Caesar’s crossing the Rubicon. The
historian, then, should not forget “that the event was an action, and that his [sic] main
task is to think himself into this action, to discern the thought of its agent” (p.213).
Collingwood’s approach to historical explanation as a unity of thought and action is not
only different from but opposed to its quantification through graphs and statistics.

By granting considerable scope for the qualitative study of history, however,
Whitehead avoids a reductionism at odds with his subsequent critique of the Fallacy of
Misplaced Concreteness. The power of the “high abstractions” of mathematical physics
has misled scientists and others into believing that they capture the whole of reality
instead of just part of it (1953, p.55). At the same time, scientific materialists disregard
as irrelevant, or even unreal, the concrete (or immediate) experience of those who feel the
natural force of gravitation or the economic effects of the rising price of bread. The result has been to exclude value from both the natural and human worlds by expunging “what is most important and basic in our lives … our concrete bodily experience [which] connects us to the world, and is thus the base for all inquiry” (Thompson, 1997, pp.222, 221; italics in the original). As Whitehead went on to argue, this fallacious reasoning has been catastrophic for both philosophy and society, because “the famous mechanistic theory of nature … [as] the orthodox creed of physical science” has led to a picture of nature as inert, valueless, disconnected bits of matter (“merely the hurrying of material, endlessly, meaninglessly”), which in the economic realm “has directed attention to things as opposed to values” (1953, pp.50, 55, 54, 202).(3)

Whitehead’s subsequent reaffirmation of experience and the way in which wisdom can increase its value by acting as a guide to both knowledge and action provides evidence of a certain consistency between his educational philosophy and his critique of the Fallacy of Misplaced Concreteness. By the time he wrote “The Rhythmic Claims of Freedom and Discipline” in 1923, he conceived of wisdom as the unity of knowledge, value, experience, and freedom:

Now wisdom is the way in which knowledge is held. It concerns the handling of knowledge, its selection for the determination of relevant issues, its employment to add value to our immediate experience. This mastery of knowledge, which is wisdom, is the most intimate freedom obtainable. (1957b, p.30)

As a major goal of education, wisdom increases the value of the learner’s experience by guiding the ways in which s/he approaches knowledge with a view to selecting how best to use it. This process enhances the life-range of her experience, its comprehensive understanding, by balancing the claims of freedom and self-discipline in an integrative manner. Interestingly, his emphasis upon wisdom and the kind of judgment it involves
are elements in contemporary approaches to mathematics education which encourage learners to be creative in the application of mathematical ideas (Ernest, 2000, p.235). These are questions to which I now turn.

5. Mathematics in a Liberal Education

5.i. Usefulness

Whitehead’s belief in the importance of mathematics was explicit not only in *An Introduction to Mathematics* but in a lecture on “The Mathematical Curriculum” delivered in 1913. Here his preoccupation with the “inclusion of mathematics in a liberal education [which] is to train the pupils to handle abstract ideas” was matched by his aversion to their becoming “recondite.” By this he meant that abstract ideas are “apt to destroy the utility of mathematics in liberal education” because of their “highly special application … [which] rarely influence[s] thought” (1957b, pp.80, 78).(4) Hence there is a need to avoid over-specialization in schools and allow students to practise simple examples that demonstrate the relationship between mathematical ideas and practical ways of thinking. The utility of geometry through the use of maps and surveying enables them to calculate the area of a field or construct the map of a small district is a case in point (pp.10-11). As advances in science and technology brought about rapid change in the modern world, the importance of mathematics would grow as the need to understand them increased.(5) A liberal education should take all of this into account and enable students to gain familiarity with the main ideas of the relations of number, quantity, and space, providing insight into the constructive ways in which they can be used (p.80).

But an emphasis upon usefulness does not mean that Whitehead considered mathematics as part of a narrow vocational training. Rather, a liberal education and its
mathematical component are useful in a more fundamental sense: It should enable students to apply ideas to their own practical lives, strengthening both their understanding and their capacity to act in creative ways. Whitehead’s account is a departure from the conventional idea of a liberal education predominant during the 19th and early 20th centuries in Britain as a rather elitist activity, one in which the study of the classics, in particular, was considered the best preparation for Christian gentlemen to run an empire. Like many others, he recognized the need for a different emphasis, one capable of enlightening students, both male and female, for life in a rapidly changing world.(6)

5.ii. Leisure

In addition to usefulness, Jean-Pascal Alcantara (2009) identifies two further characteristics of Whitehead’s conception of a liberal education, which cast light on his conception of the teaching of mathematics. The first of these is a particular orientation to temporality, or the passage of time. The Greek concept of ωξοδή, or Latin *otium*, is relevant, since, according to Alcantara, “this term originally means a ‘liberal manner,’ namely not ‘being rushed or ‘concerned by time’” (p.132). An echo of this idea can be found in Whitehead’s injunctions: “Do not teach too many subjects,’ and again ‘What you teach, teach thoroughly’” (1957b, p.2). Since he went on to discuss the teaching of science and logic, it is reasonable to assume that the idea of not rushing through the curriculum includes mathematics, especially given its abstract nature. In the lecture, titled, “The Place of Elementary Mathematics in a Liberal Education,” he proposed that the mathematical curriculum be more closely related to the needs of students, because “lack of time is the rock upon which the fairest educational schemes are wrecked” (1965, p.38, my italics).
The same principle applied at the university level with regard to introductory courses in geometry and algebra, where “great care should be taken not to overload the mind [of the student] with more detail than is necessary for the exemplification of the fundamental ideas” (1958, p.187). But it is not only students who needed the time to appreciate abstract ideas but also professors, whose work could only flourish in a relaxed and open atmosphere. In a later lecture, titled “Universities and their Function,” Whitehead asserted that a faculty with “the combination of imagination and learning normally requires some leisure, freedom from restraint, freedom from harassing worry, [and] some variety of experiences.” Where these conditions are not met, rather than inspiring imaginative, intellectual relationships with their students, there is a danger of producing “a faculty of very efficient pedants and dullards” (1957b, pp.97, 99).

5.iii. History of Mathematics

There is a further aspect of the orientation towards time which is especially important with regard to the teaching and learning of mathematics for Whitehead. An appreciation of its history and the ways in which mathematical ideas have developed are necessary conditions for a general understanding of the discipline. This insight provides a pedagogical tool which can be used for changing potentially “recondite,” or inert, ideas into abstract ideas which have meaning to students because they relate to their concrete experience. In An Introduction to Mathematics, every chapter includes stories of mathematicians whose exploits bring the discipline’s abstractions to life and situate them in their historical context. There is no better example than the chapter on “Methods of Application,” which reflected his interest in applied mathematics.
Here Whitehead drew attention to the following general principle: “The conclusion of no argument can be more certain than the assumptions from it starts” (1958, p.16). Since the laws of nature are the premises from which mathematical calculations about natural events are deduced, they too can be questioned. An example of this general principle is Newton’s formulation of The Law of Gravity, allegedly sparked by his observation of a falling apple. First of all, this dramatic breakthrough could not have occurred without “the mathematical habit of mind and mathematical procedure” having developed over many centuries, a process without which “Newton could never have thought of a formula representing the force between any two masses at any distance” (1958, p.17). Galileo’s work with regard to the ideas of force, mass, and distance, in particular, set the stage for Newton’s theoretical explanation of the fall of an apple and the motions of the planets. Second, with regard to the principle in question, subsequent advances in the new physics show just how limited the premises of Newtonian mechanics were. As Whitehead put it in Process and Reality:

The fate of Newtonian physics warns us that there is a development in scientific first principles, and that their original forms can only be saved by interpretations of meaning and limitations of their field of application – interpretations and limitations unsuspected during the first period of successful employment (1957a, p.13)

The case of Newton illuminates the need to recognize the ambiguity and limited applicability of first principles, however clear and distinct they may appear at the time.

This is a lesson which Whitehead wished students to learn from an early age.

The history of the science of electromagnetism and its gradual use of mathematical methods provides another example of the practical uses of abstract ideas. Although the Chinese made use of the compass needle more than 3,000 years before its
introduction into Europe, it was not until the late 18\textsuperscript{th} century that two Italians, Galvani and Volta, discovered the electric current and its possibilities for the modern science of electromagnetism. As mathematical ideas were refined, advances in the science were made by various scientists, including Clerk Maxwell, who suggested that “the vibrations which form light are electrical” (Whitehead, 1958, p.21), an insight which contributed to the development of quantum mechanics. The purpose of Whitehead’s “rapid sketch” was to show,

… how, by the gradual introduction of the relevant theoretic ideas … a whole mass of isolated and even trivial phenomena are welded together into one coherent science, in which the results of abstract mathematical deductions, starting from a few simple assumed laws, supply the explanation to the complex tangle of the course of events.” (p.22)

The theoretical assumptions articulated as laws of nature provide the basis for mathematical deductions capable of providing coherence to, and an explanation of, the complex events comprising the science of electromagnetism.

Turning to mathematical physics, Whitehead acknowledged that non-scientists, including Chaldean shepherds as well as government officials in Mesopotamia and Egypt, were catalysts in its development. Their observations of “a regularity of events” in the skies or when engaged in measuring the land produced “detached speculations” without any real understanding of their “interconnection” (1958, p.22). The geometry necessary for determining the exact regularity of the solar system may have grown out of the practice of land-surveys, but it was not until the work of Archimedes, “who combined a genius for mathematics with physical insight, [and who] must rank with Newton … as one of the founders of mathematical physics” (p.23). The famous story about Archimedes having been asked by the king of Syracuse if his crown was pure gold, leapt
from his bath and ran through the streets, shouting “Eureka!” when overjoyed at discovering how to solve the problem “ought to be celebrated as the birthday of mathematical physics” (p.24). He had discovered that a body when immersed in water is pressed upwards by the surrounding water with a resultant force equal to the volume of the water it displaces, a fact which can “be proved theoretically from the mathematical principles of hydrostatics and can also be verified experimentally” (p.24). It may well have been the first occasion on which mathematical ideas were applied to physics, thereby exemplifying “the method and spirit of science” in general (p.25).

In terms of teaching the science of mathematics, all these historical examples are capable of bringing abstract ideas to life. From Archimedes leaping from his bath to Galileo, who “rediscovered the secret, known to Archimedes, of relating abstract mathematical ideas with the experimental investigation of natural phenomena” (p.27), and dropped different weights from the leaning tower of Pisa to show that gravity operated in a uniform manner (de Berg, 1992, p.79); to the acrimonious debate between Newton and Leibniz about who discovered the differential calculus. More important than the dispute between them is how it illustrates that “the subject had arrived at a stage in which it was ripe for discovery, and there is nothing surprising in the fact that two such able men should have independently hit upon it” (Whitehead, 1958, pp.163-4).

Whitehead’s goal throughout is to provide a historical narrative about the development of mathematics by means of concrete examples designed to help in the teaching and learning of the discipline. Is it reasonable, therefore, to suggest that he laid the ground for current debate about the teaching of mathematics? After all, he did propose that: “Another way in which the students’ ideas can be generalized is by the use
of the History of Mathematics, conceived … as an exposition of the general current of thought which occasioned the subjects to be objects of interest at the time of their first elaboration.” By proposing to humanize mathematical pedagogy by teaching its history, Whitehead believed that “it is the very subject which may best obtain the results for which I am pleading” (1957b, p.84).

In making this argument, he anticipated some of the modern scholarship on the history and philosophy of science and mathematics teaching. While Ivor Grattan-Guinness (2010) claims that Whitehead “did not advocate teaching mathematics via the direct use of older sources” (p.258), he does acknowledge there to be a loose connection with the work of such contemporary organizations as the International Study Group on the Relations between History and Pedagogy of Mathematics, which advocates the use of history in explaining the motivations and purposes of mathematical theories. Kevin de Berg (1992) does not mention Whitehead, but his adoption of the study of historical profiles in the teaching of mathematical components of the pressure-volume laws of gases (e.g. Boyle’s law) helps students to understand abstract ideas, because it “addresses the dynamic emergence of the scientific law and provides opportunities for increased student participation” (p.86). Paul Ernest (2000) goes further, suggesting informal connections between Whitehead’s use of examples from the history of mathematics and several contemporary pedagogical approaches.(8) One of these is particularly striking: “Studying cultural contexts of mathematics and mathematical ideas can show the relevance and origins of mathematics in both proximal and distant contexts” (p. 235). The multicultural history of mathematics is something to which Whitehead ascribed considerable importance. Unlike some Western thinkers of his time, he acknowledged
the debt owed to Eastern mathematicians, and believed that students should learn as much
about the ways in which abstract ideas are shared across national boundaries as possible:

    The really inspiring reflection suggested by the history of mathematics is the unity
of thought and interest among men [sic] of so many epochs, so many nations, and
so many races. Indians, Egyptians, Assyroians, Greeks, Arabs, Italians,
Frenchmen, Germans, Englishmen, and Russians have all made essential
contributions to the science (1958, p.164). (9)

Mathematicians depend upon the advancement and dissemination of shared knowledge
by others working in the field, whatever their nationality.

    At the same time, Whitehead had nothing to say about contributions to
mathematics made by Indigenous peoples in the Americas. The Aztec and Mayan
capacity to plot the movement of the stars and the planets and their construction of
calendars are obvious examples. Circumpolar peoples’ use of mathematics has recently
been documented (Sriraman & Fyhn, 2011) through collaborative research between
elders among the Yup’ik people in Alaska and experienced Yup’ik teachers, for example.
The research has shown that everyday activities like weaving a ceremonial headdress
involve the use of a variety of mathematical concepts (Lipka, Andrew-Ihrka, &Yanez,
2011, pp.165-170). The application of these concepts include proportional measuring
based on the use of a specific body measure called “knuckle length,” which enables the
weaver initially to generate a square representing the spiritual basis of Yup’ik cosmology
and then, by means of a series of folds, a circular pattern representing the Four
Directions. While the researchers are “speculative” (p.157) in their claims, they have
found that this practice generates an understanding of the following mathematical
concepts: “geometrical constructions, verification of geometrical properties, [and]
relationships between geometry, numbers and number theory (through the equal
partitioning by the symmetry folding process)” (p.170). The use of a theoretical  
framework, known as Mathematics in a Cultural Context (MCC), which has utilized such  
examples for almost a decade with 10,000 students in 20 Alaskan school districts, has  
shown “statistically significant improvements in math performance” (p.158). (10)  

5.iv. Mathematics and Technical Education  
The synthesis of liberal and technical education is another characteristic which Alcantara  
correctly ascribes to Whitehead (2009, pp.135-7). The goal was to overcome the  
opposition between intellectual and manual learning. This innovative proposal afforded  
mathematics pride of place as the core of the framework for reform, since it provides a  
platform for integrating the sciences and the humanities with a technical education that  
enables students to create beautiful objects with their hands. And technical education is  
itself important as capable of coordinating “head-work” and “hand-work,” theory and  
practice (1957b, pp.49-50).  

Whitehead began his 1917 lecture, “Technical Education and its Relation to  
Science and Literature,” by stating that the issue is “a very burning question among  
mathematical teachers; for mathematics is included in most technological courses”  
(1929b, p.43). As governor of the Borough Polytechnic Institute in London, he was  
familiar with the curricular changes taking place and aware that mathematics was a key  
element in the process. And as chair of the Delegacy administering Goldsmith’s College,  
a major institution for students in art and design as well for teacher education, Whitehead  
could foresee the possibilities for integrating technical education with mathematics and  
science, on the one hand, and literature and the arts, on the other (Hendley, 2010, p.78).
He defined technical education quite broadly as “a training in the art of utilizing knowledge for the manufacture of material products,” for which are needed “manual skill, and the coordinated action of hand and eye, and judgment in the process of construction.” The process of “hand-craft” involves “a reciprocal influence between brain activity and material creative activity” in which “the hands are peculiarly important,” and enables students to put their ideas into practice by making objects with increasing dexterity. Hence, technical education embodies a fundamental principle to which Whitehead subscribed, namely, “If you want to understand anything, make it yourself” (1957b, pp.49-53). The need for craftspeople who create beautiful objects in wood, metal, and other media as well as farmers and cooks is overwhelming in the modern world (pp.55-6).

In order for hand-craft to be successful, however, some scientific knowledge is required in the form of an understanding of “those natural processes of which the manufacture is the utilization.” The scientific education Whitehead had in mind involved both “a training in the art of observing natural phenomena, and in the knowledge and deduction of laws concerning the sequence of such phenomena.” Mathematics plays a key role in learning to deduce laws of nature based on accurate observations, and provides a theoretical base for the activities of technical education. At the same time, technical education can overcome “the [kind of] narrow specialism” too often found in “a study of science” (1957b, 50, 49) by keeping ideas fresh in students’ minds.

The full integration of the curriculum is only possible, however, with the inclusion of literary studies, or “the study of language,” its structure, techniques of verbal expression, and relationship to intellectual feelings. The language of poetry or prose
appeals to “the sense organs” and fosters their development as a channel for the expression of feeling in aesthetic and constructive ways. Analogously, it is “bodily feeling[s] … focused in the eyes, the ears, the voice, the hands” which provide the “reciprocal influence between brain activity and material creative activity” at the base of technical education (1957b, 49-50). On the one hand, the artistic use of language emancipates the thoughts and feelings of the speaker; on the other, the bodily feelings of the student are liberated though the creative practice of the plastic arts. The two forms of education complement one another, which is why Whitehead claimed that: “geometry and poetry are as essential as turning lathes” (1957b, p.45). In crafting objects with one’s hands, mathematical knowledge combines with a poetic appreciation of the worth of one’s work.

Harold Entwistle (1975) claims that Whitehead was arguing for what has since been called a “spiral curriculum” (p.115) in which there is an alternating emphasis upon the literary, the scientific, and the technical. Just as the cycles of romance, precision, and generalization constitute the general rhythm of education, so “the problem of education is to retain the dominant emphasis, whether literary, scientific, or technical and without loss of coordination to infuse into each way of education something of the other two” (Whitehead, 1957b, p.54). This alternating emphasis, or rhythm, integrates all three spirals, producing what Whitehead called a “seamless coat of learning” that imparts “an intimate sense for the power of ideas, the beauty of ideas, and for the structure of ideas” (pp.11-12). Nor should technical education be “conceived as a maimed alternative to the perfect Platonic education” since one of “the evil side[s] of the Platonic culture has been its total neglect of technical education” (p.54).(11)
For Whitehead (1957b), the aesthetic emotions embodied in technical education provide students with “the sense of value, the sense of importance … the sense of beauty, the aesthetic sense of realized perfection” as their work is imbued by the integration of head-work and hand-work (p.40). Mathematics can also share in this process of releasing the aesthetic emotions at the core of such creative activities, provided that it plays a connecting role between the sciences, the humanities, and technical education.(12) As we have seen, science education relies upon mathematics to deduce laws from a tangle of observations; students of technical education use mathematics in making measurements when constructing objects out of wood, metal, and other materials like clay; but what relevance does it have for literature and art? The following suggestion made by Alcantara (2009) is helpful: “Whitehead’s proposal for reforming mathematical training looks to be two-sided, emphasizing, at the same time, the most beautiful abstract theories as well as its practical usefulness” (p.136). Precisely because of the beauty inherent in mathematics - the abstract formulae and proofs it develops, its usefulness in broadening students’ experience, the laws of nature and the patterns it uncovers, the applications to society it makes possible, and its integrative role in technical education – there is a close relationship with the humanities and arts, which is capable of fostering a balanced understanding too often missing in the modern world.(13) A colleague of mine who teaches Irish poetry often writes the first couple of lines of a poem on the board, and then asks students what they think the next line is. More often than not, they construct it correctly and go on to do the same for much of the poem, illustrating his point that rhythm, pattern, and structure are integral to their experience. When conjoined in this
manner, poetry and mathematics can strengthen the capacity and interests of students, so that they feel equipped to bring more beauty into the world (Cobb, 1998, pp.106-7).

6. The Rhythm of Life

In *Art as Experience* (2005), Dewey argued that everyday experiences like eating a meal, going for a walk or even having an argument with a loved one give rise to aesthetic moments as the basis of art and aesthetic appreciation. Life, he argued, “is no uninterrupted march or flow. It is a thing of histories, each with its own plot … each having its own particular rhythmic movement” (p.37). This rhythmic movement, which constitutes the basis of the aesthetic in our experience, is rooted in a primordial connection with the rhythms of nature to which art is especially sensitive:

The first characteristic of the environing world that makes possible the existence of artistic form is rhythm. There is rhythm in nature before poetry, painting, architecture and music exist. Were it not so, rhythm as an essential property of form would be merely superimposed upon material, not an operation through which material effects its own culmination in experience. (p.153)

Without natural rhythm, artistic form would be no more than an imposition upon material rather than a way to express the full realization of human experience. Furthermore, the “larger rhythms of nature,” Dewey wrote, permeate human existence - “dawn and sunset, day and night, rain and sunshine, are in their alternation factors that directly concern human beings.” Indeed, “the circular course of the seasons affects almost every human interest” (p.153), the growth of agriculture, in particular.

This alternating rhythm of nature was something to which Whitehead had drawn attention more than twenty years earlier in *An Introduction to Mathematics*. “Periodicity” as the alternating rhythm of life is a brute fact to which mathematics must be attuned as a scientific instrument:
The whole of Nature is dominated by the existence of periodic events … [and] one of the first steps necessary to make mathematics a fit instrument for the investigation of Nature is that it should be able to grasp the essential periodicity of things (pp.121, 127).

The examples he gave of periodicity were similar to Dewey’s. The rotation of the earth produces day and night as well as the seasons, “and [it] imposes another periodicity on all the operations of nature … [including] the phases of the moon” (p.121). Whitehead also believed that periodicity embodies both sameness and difference, repetition and contrast.

(14) Periodic events like the days of the month are “recurrences of the same event” produced by “the rotation of the earth” and are “different from the preceding days.” The reason for this contrast lies in our ability to abstract from the particularity of each day, so that “the distinction in properties between two days becomes faint and remote from practical interest.” This process of abstraction from the concrete events of one day then leads to the general idea of a day “as a recurrence of the phenomenon of one rotation of the earth” (p.121). The danger of abstracting from concrete events in this way is to lose sight of the dynamic relationships obtaining among them.

(15) How, then, to avoid this?

In *The Concept of Nature*, lectures delivered in 1919 at Trinity College Cambridge, Whitehead utilized the concept of spatial and temporal extension to understand the dynamic continuity of nature:

The continuity of nature arises from extension. Every event extends over other events, and every event is extended over by other events … The concept of extension exhibits in thought one side of the ultimate passage of nature. This relation holds because of the special character which passage assumes in nature (1971, pp.59, 58).

The continuity, or periodicity, of events in nature (such as one day succeeding another), arises from their extension. And extension is to be understood as the overlapping of events in both space and time which is, according to Whitehead, “the fundamental fact …
[of] the passing of nature, its development, its creative advance” (p.34). The dusk which follows the sunset overlaps with both day and night, connecting them in a creative process of contrasting light and dark. Without this kind of passage in nature, scientific materialism would hold true, according to which “nature is an aggregate of material … [which] exists in some sense at each successive member of a one-dimensional series of extensionless instants of time” (p.71). If nature were made up of lifeless, separate bits of matter, there would be no duration through time, no flow, simply isolated, instantaneous points in a temporal (or spatial) series without extension.

Moreover, the periodicity of nature is an experience gained through the functioning of our own bodies, as Whitehead further observes in An Introduction to Mathematics. “Our bodily life,” he wrote, “is essentially periodic. It is dominated by the beatings of the heart, and the recurrence of breathing. The presupposition of periodicity is indeed fundamental to our very conception of life” (1958, p.122; my italics). Not only are our own lives dependent upon the periodic beatings of the heart and our recurrent breathing, but our very conception of life as flowing through time “is itself dependent on the observation of periodic events” (p.125). There are, however, distinctive characteristics to the periodicity of human life: “Broadly speaking, all recurrences depending on living beings, such as the beatings of the heart, are subject in comparison with other recurrences to rapid variations” (p.123). These rapid variations can occur in various situations: when we are under stress, or when we run, walk or do Tai Chi. As Frederic Bisson (2009) suggests, the rhythmic beatings of the heart issuing from the cadences of the atrial and ventricular chambers produce “[un] dephasage infinitesimal [qui] produit dans l’intervalle le battement ternaire des ‘bruits de coeur’” (p.27). There is
a tiny phase, an interval during which the compound beating of the heart is produced as
the sounds we hear. Ian Winchelsey also suggests that:

Whitehead must have been thinking of the contraction and expansion phases in
order to have the picture of [the heart functioning periodically] ... The heart has
four chambers and has essentially two main states, one in which it is contracting
and one in which it is expanding. The contraction phase is referred to as ‘systole,’
the expansion phase is referred to as ‘diastole.’ (Email to the author, April 13,
2012).

In other words, systolic contraction and diastolic expansion comprising the major phases
of the heart do occur periodically in an overall process of repetition in which each
contrasts with the other in a distinctive rhythm of the human pulse.(16)

There is, I believe, a relationship between Whitehead’s account of the rhythm of
the human heart and the rhythmic cycles of growth at the core of his educational thought.
Periodicity and variation as the basis of life recur as key concepts in “The Rhythm of
Education,” originally published as a pamphlet in 1922:

Life is essentially periodic. It comprises daily periods, with their alternations of
work and play, of activity and of sleep, and seasonal periods, which dictate our
terms and our holidays; and also it is composed of well-marked yearly periods.
These are the gross obvious periods which no one can overlook. There are also
subtler periods of mental growth, with their cyclic recurrences, yet always
different as we pass from cycle to cycle, though the subordinate stages are
reproduced in each cycle. That is why I have chosen the term ‘rhythmic’ as
meaning essentially the conveyance of difference within a framework of
repetition. Lack of attention to the rhythm and character of mental growth is a
main source of wooden futility in education. (1957b, p.17)

Here Whitehead situated education in the broader context of life, whose alternating
patterns reflect the need for rest, sleep, work, and play. The same patterns are to be
found in the larger seasonal and yearly periods which determine the timing of the school
year. But there are more subtle periods of mental growth which also incorporate the
pattern of the seasons, as they recur and contrast with one another in similar fashion. The
different cycles overlap with one another ("during the stage of precision, romance is the
background” p.34; my italics), and their interrelationship underlines a central point;
namely, that the rhythmic character of growth is a process in which different cycles occur
within a recurrent framework. Hence, romance as the joy of discovery is the initial cycle
without which precision will be barren; and similarly precision is necessary for
generalization to bear fruit. If educators fail to understand these periodic and varied
patterns of growth, the result will be what he called a “wooden futility.” (17)

If indeed there is a connection between the periodicity of the human heart beat
and the alternating rhythm of education, what would this tell us about the latter? When
writing about the importance of romance, Whitehead described it as a lengthy phase in
which the student should be allowed “above all … plenty of independent browsing amid
first-hand experiences, involving adventures of thought and of action” (1957b, p.33). In
contrast, there is a danger that precision will kill this sense of adventure, and so “in the
way of caution, there is such a thing as pushing on … in respect to precise knowledge,
the watchword is pace, pace, pace. Get your knowledge quickly, and then use it” (pp.34,
36). Precision, then, is to be a relatively short phase in the overall cycle of learning. By
way of contrast, generalization moves closer to the rhythm of “the discursive adventures
of the romantic stage, with the advantage that … [the student’s] mind is now a disciplined
regiment instead of a rabble … An education which does not begin by evoking initiative
and end by encouraging it must be wrong” (p.37). Generalization as the process of
integrating general principles with one’s concrete experience is an open ended phase, one
which may never be complete.
Given this textual evidence, the speculative hypothesis I wish to advance is that the rhythm of the human heart (Long – Short – Long) is analogous to the alternating rhythm of the cycles of growth (Long, for Romance – Short, for Precision – Long, for Generalization). The significance of this interpretation is that Whitehead’s theory of learning is organic in a fundamental way, since it reflects the very pattern of life in nature, which he had been at pains to argue for in *An Introduction to Mathematics*. More specifically, it shows that the process of human learning corresponds to the very metabolism of the beating heart which makes an individual’s life possible. Without the latter, we would die; and without education, the range of our life-value as the capacity to learn more comprehensively would be stunted (McMurtry, 1998, p.298).

This is why Whitehead emphasized the importance of “the art of life” as the goal of education. The art of life enables the kind of creative growth he had in mind:

Education is the guidance of the individual towards a comprehension of the art of life; and by the art of life I mean the most complete achievement of varied activity expressing the potentialities of that living creature in the face of its actual environment. (1957b, p.39)

Education, then, should guide the individual towards an understanding of the diverse activities, which express the full potentialities which s/he can attain in the context of the environment with which s/he is faced. Learning the art of life enables individuals to realize their full capacities by understanding the world in an aesthetic, joyful, and precise way. This process is best realized when the rhythm of growth is in accord with “an alternation of dominance … which constitutes the cycles” of learning, one which accords with the primordial beating of the human heart (p.28).

7. Conclusion
When writing of the goals of liberal education in universities today, Canadian historian Paul Axelrod argues that intellectual creativity, a combination of intellectual breadth and specialized knowledge, as well as the comprehension and respect for diverse ideas and cultures:

... can and should be integrated into scientific, technical, and professional education ... [since they] can both enrich and profit from academic initiatives that seek to integrate (rather than isolate) the arts, the sciences, vocational training, and the professions (2002, p.35).

This same vision of liberal education as a powerful force for integrating the fragmented curriculum of universities and schools is one which Whitehead was advocating one hundred years ago.

His emphasis on a liberal education in which mathematics plays an important role is of more than historical interest. There are, as I have shown, several ways in which his account resonates with the aspirations of educators today. Introductory courses to mathematics even at the university level should avoid technical details and concentrate on general ideas and their significance in the world.(19) Otherwise, these same ideas are in danger of becoming inert, and students bored by the whole process. There should also be sufficient opportunity to explore the abstract ideas of mathematics in romantic fashion, allowing students the leisure to peruse them at their own pace. The relevance of this proposal to a world which has lost sight of the root meanings of leisure -- *skhole* (Greek) and *schola* (Latin) -- is clear. The history of mathematics and science taught as an integral part of both disciplines is capable of bringing ideas and practices to life as students recognize the contributions of men and women from many nations who have created such knowledge.(20) This pedagogical approach to mathematics has been taken
up in recent years by various professional organizations and curriculum planners throughout the world. (21)

Furthermore, in an age such as ours when an emphasis on vocational training for the corporate market has almost drowned out any discourse about broader, humanistic conceptions of education (Woodhouse, 2009), a synthesis such as Whitehead proposes between intellectual and practical work could provide an alternative for students, whatever their interests, to exercise their capacities for learning; namely, a form of education in which mathematics plays a pivotal role in uniting science, the humanities, and technical education, or learning to craft objects of beauty with one’s hands. The very rhythm of life - its periodicity, or the process of succession in nature punctuated by contrasting events in the sequence of day and night, for example – can be discerned by the methods of mathematics. And this same rhythm characterizes not only the human heart beat but the cycles of growth which constitute human learning, especially where students come to appreciate the art of life as the fulfillment of their diverse potentialities. All of this suggests a new vision, one in which students are educated to appreciate the humanizing influence of mathematics by being exposed to key ideas coupled with a few simple details and their relationship with other disciplines.

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Notes

1. Jean-Pascal Alcantara (2009) claims that during the Renaissance a revival of Greek and Latin humanities combined with the new conception of human beings as free from the traditions of the past created a conception of liberal education which had competing streams to it (pp.128-130).

2. Elsewhere, I have shown there to be a relationship between the rhythmic cycles of growth and the “characteristics of life” in Whitehead’s philosophy of organism (Woodhouse, 1995, pp.348-353).

3. For more on the Fallacy of Misplaced Concreteness, see Charles Birch (1988) and for a seminar on the same subject, see H. Edward Thompson (1997), Mark Flynn (1997), Robert Regnier (1997), and Howard Woodhouse (1997).

4. This quotation anticipates to some extent the Fallacy of Misplaced Concreteness and its critique of “the high abstractions” underpinning 17th century mechanistic materialism and their exclusion of human experience.

5. In a lecture delivered to the Harvard Business School and later published in Adventures of Ideas (1961) Whitehead claimed that Foresight, based on philosophical understanding and generalization, is the capacity best suited for understanding the rapidly changing modern world of commerce (pp.97-8).

6. These changes in the approach to liberal education in the early 20th century are documented by Paul Axelrod (2002, pp.22-4). An alternative account to liberal education as exhibiting the value of “useless knowledge” was given by Bertrand Russell (2006). For a commentary on Russell’s views, see Woodhouse (2006).

7. At the same time, Whitehead recognized that in practice science advances despite imprecise calculations and laws that are open to question – “so, after all, our inaccurate laws may be good enough” (1958, p.16). In Process and Reality (1929a), he reasserted that it is the “general success” of first principles, not their “peculiar certainty or initial clarity,” which is the goal of rational thought, since “even in mathematics the statement of the ultimate logical principles is beset with difficulties, as yet insuperable” (p.11). While metaphysical first principles can always be questioned, they are open to human experience: “There is no first principle which is in itself unknowable, not to be captured by a flash of insight … The elucidation of immediate experience is the sole justification for any thought; and the starting point for thought is the analytic observation of components of this
experience.” As a result, “metaphysical first principles can never fail of exemplification. We can never catch the actual world taking a holiday from their sway” (p.11). Ian Winchester (2000) observes that Whitehead was engaging in a critique of the limitations of the empirical or scientific method: “Because of the omnipresence of the metaphysical there are limits to the employment of the empirical method here. Thus any account of meaning that limits thought to the empirical is bound to break down just as it approaches the metaphysical” (p.297). For a critique of Newton’s attempt to exclude hypotheses from “experimental philosophy,” see Toni Carey (2012).

8. Paul Ernest (2000), like Kevin de Berg (1992), underlines the ways in which mathematics has been vital in the development of quantum mechanics, as does Amir Aczel (2002, p.252). Aczel also shows how the history of the process of entanglement proposed by mathematical physicists is an unfolding story which, as chance would have it, also illustrates the importance of internal relations and feelings in Whitehead’s ontological sense (2002, pp.xiii, xv, 1, 250-2).

9. According to one historian, “the intellectual renaissance of the Latin West in the twelfth and thirteenth centuries would not have been possible without the advances made by the scholars of Islam. Their translations, commentaries, and original speculation on a wide variety of subjects were among the cornerstones of the European curriculum” (Domonkos, 1977, p.9, cited in Axelrod, 2002, p.14). See also, Haskins (1960, pp.4-5).

10. Current research utilizing the same approach of MCC is also being conducted with the Sami people in Norway (Fyhn, Sara Eira, Sriraman, 2011). As my colleague Ed Thompson has pointed out in conversation, there are two ways in which the study of indigenous mathematics may prove valuable. First, to provide new insights into both the theoretical framework and teaching of mathematics; second, to enable the well established mathematician to engage in philosophical reflection about the question, “What is it that all of us mathematicians are actually doing?” For examples of mathematics education in a Whiteheadian vein, see Robert Brumbaugh (1992), Leslee Pelton (1995), and Murray Guest (2010).

11. The Platonic belief in “disinterested intellectual appreciation” as the goal of education should be replaced by an emphasis on “action and our implication in the transition of events amid the inevitable bond of cause to effect” (Whitehead, 1959b, p.47). Students learn to bring about change by creating objects of beauty through a combination of thought (head-work) and action (hand-work). They thereby come to appreciate the importance of “causal efficacy,” or “the ‘withness’ of the body … that makes the starting point for our knowledge of the circumambient world” (Whitehead, 1959a, p.81). The bodily feelings expressed in the unity of mental and manual labour provide a direct epistemological connection between the learner and reality.
12. For a more detailed analysis of the importance of art and craft in overcoming alienated labour in Whitehead’s educational philosophy, see Adam Scarfe and Woodhouse (2008, pp.195-6).

13. G.H. Hardy remarked that “A mathematician, like a painter or a poet, is a maker of patterns … The mathematician’s patterns, like the painter’s or the poet’s, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test … (1967, pp.84-5; cited in Maryvonne Hallez, 1992, p.313). As Ronny Desmet points out: “According to Whitehead, mathematics is the study of relational structures or patterns. The emergence of mathematics involves the direct pattern recognition that is proper to our sense perception … [which] is inseparably tied to our feeling of space, and because our space-intuition is so essential an aid to the study of geometry, it seems as if geometry cannot be part of pure mathematics” (2010a, p.369). Desmet goes further, arguing that, “his [Whitehead’s] ultimate drive was … to unify the mathematical structures underlying the analogical reasonings that constitute the art of physics, an art which his Cambridge training impressed on him” (2010b, p.121). Whitehead himself (1958) calls attention to the importance of spatial intuitions despite their logical independence from the mathematical science of geometry (pp.180-1).

14. Dewey (2005) sounds very much like Whitehead when writing: “What is not so generally perceived is that every uniformity and regularity of change in nature is a rhythm. The terms ‘natural law’ and ‘natural rhythm’ are synonymous … Mathematics are the most generalized statements conceivable corresponding to the most universally obtaining rhythms. The one, two, three, four, of counting, the construction of lines and angles into geometric patterns, the highest flights of vector analysis, are means of recording or of imposing rhythm” (p.155). Bernie Neville (2009) analyzes biological rhythms in terms of monthly, daily, and ultradian phases (pp.63-4).

15. The concept of a periodic function is Whitehead’s (1958) preferred method of expressing these formal abstractions on the basis of “a sum of sines … called the ‘harmonic analysis’ of the function … [which produces a] process of gradual approximation” capable of analyzing such events as the relationship between “the tide-generating influences of one ‘period’ to the height of the tide at any instant” (pp.142, 143). Later, when writing of the development of science in the 16th and 17th centuries (1953), he states that: “The birth of modern physics depended upon the application of the abstract idea of periodicity to a variety of concrete instances. But this would have been impossible, unless mathematicians had already worked out in the abstract the various abstract ideas which cluster round the notions of periodicity … Then, under the influence of the newly discovered mathematical science of the analysis of functions, it [trigonometry] broadened out into the study of the simple abstract periodic functions which these ratios exemplify” (p.31). Commenting on Whitehead’s use of periodic functions in expressing events like day and night, the seasons as well as tides, Ian Winchester
writes: “Both sines and cosines can be portrayed as an undulating line above and below passing regularly through a straight line, usually a horizontal one. It is not obvious to me that all of these periodic regularities are best portrayed this way, but in general they can be so portrayed. It depends on what one plots. Day and night, for example, have to do with the amount of sunlight daily which could be so portrayed, though once one has passed the "no sunlight" point one would have to think of darkness, I suppose, which is a bit of a strain. In northern and southern regions the sine (or cosine) curves would be shifted up or down relative to the line. The seasons are related to the relation of the sun relative to the axis of the earth, and at the equator the daily rhythm of day and night is exactly twelve hours, and varies north and south from there, while the seasonal variation, which disappears at the equator picks up on both sides until one has maximum variation at the poles. So again a kind of sine or cosine function would be possible to portray it annually. The tides would, I suppose, be a matter of portraying the height of the tide hourly using some neutral base point so that there would be high tide and low tide, high above the line where the slope of the sine curve is zero and the low time similarly below the line where the slope of the sine curve is also zero.” (Email to author, 11 April 2012.)

16. Winchester analyzes this complex rhythm as follows: “In fact the various chambers work in a sequence and the detailed story is rather complicated, but if one concentrates on systole and diastole then one can picture the filling up or expansion phase of the heart (i.e. diastole), when it takes in blood from the wider circulatory system that has been returned to it through the venous drainage, as one of the semi-periodic movements (say above the horizontal line) and the contraction phase or systole, when it expels the blood that it has received to the rest of the body (including, in fact, itself through the coronary arteries) might be seen as the movement below the line … I have cut out the interesting details in order to see how Whitehead might have used the sine curve as a metaphor for the overall process … But the heart does function with a large major expansion and a large major contraction which enables the sine curve to plot the overall story of the pulse.” (Emails to the author April 13 & 14, 2012.)

17. This same process also provides the basis for freedom of action, provided that “individuality” is balanced with a further recognition that “coordination … of community life” is a significant aspect of students’ environment (Whitehead, 1961, p.67). Only where the individual strives to integrate the polarities of “self-development” and “the complex pattern of community life,” can the “art of life” take hold (1957b, p.39; 1961, p.67). While this kind of balance is a major goal of liberal education for Whitehead, it should also include enjoyment of the emotions, as he notes in Modes of Thought (1966): “Life is the enjoyment of emotion, derived from the past and aimed at the future. It is the enjoyment of emotion which was then, which is now, and which will be then. This vector character is of the essence of such entertainment” (p.167). For an analysis of this vectoral character of emotions, see Woodhouse (2012).
18. Only where students appreciate the beauty in nature and human artifacts, and the panoply of changing values inherent in both, will they learn the art of life, “(i) to live, (ii) to live well, (iii) to live better.” Art and aesthetic appreciation enable human beings to lead civilized lives in which they strive “towards the attainment of an end realized in imagination but not in fact” (1958, pp.4, 8). At the same time, art brings the potentiality of the imagination into the actuality of everyday life.

19. At the same time, Whitehead warned that: “We shall ruin mathematical education if we use it merely to impress general truths. The general ideas are the means of connecting particular results. After all, it is the concrete special cases which are important. Thus in the handling of mathematics in your results you cannot be too concrete, and in your methods you cannot be too general” (1957b, p.53).


References


