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Anders Johan Lexell's Role in the Determination of the Solar Parallax

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Abstract. Anders Johan Lexell (1740–1784) was a mathematician who gained considerable recognition for his scientific achievements during the century of Enlightenment. Born and educated in Åbo/Turku in the Finnish part of the Swedish Realm, he was invited as an assistant and collaborator of Leonhard Euler at the Imperial Academy of Sciences in Saint Petersburg in 1768. After Euler's death in 1783 he inherited his mentor's chair and became professor of mathematics at the Petersburg Academy of Sciences, but survived only a year in this office. One of Lexell's first tasks in Saint Petersburg was to assist in the calculations involved in the Venus transit project of 1769. Under Euler's supervision, Lexell formulated a system of modeling equations involving the whole bulk of observation data obtained from all over the world. Thus, by searching (manually) the best estimate of the parallax with respect to all available measurements made of the Venus transit simultaneously, he anticipated later statistical modeling methods. The usual method at the time consisted of juxtaposing a pair of measurements at a time and taking a mean value of all the parallax values obtained in this way. What had started as an innocent, purely academic attempt to establish the solar parallax, soon escalated into a heated controversy of international dimensions. The roles played by Jérôme de Lalande in Paris and Maximilian Hell in Vienna in this controversy are well known; Lexell's role less so. Our analysis has two aims. First, we elucidate Lexell's place in the international solar parallax controversy by making use of his published works as well as surviving parts of his correspondence. Second, we present the method used by Lexell and analyze his way of calculating the solar parallax.

1. Introduction

Anders Johan Lexell (1740–1784) is best known as a mathematical astronomer with two major achievements: he calculated that the "star" found by Herschel in 1781 moved in a nearly circular orbit around the Sun, thereby concluding that it must be a planet (i.e. Uranus), and he elucidated the very special motion of the Comet D/1770 L1, also known as the "Comet Lexell" (Grigorian & Youshkevich 1970–1980; Lehti & Markkanen 2010). His real vocation, however, was mathematics (*Encyklopädie der Matematischen Wissenschaften, 1907*, Grigorian & Youshkevich 1970–1980, Lysenko 1980). As far as the Venus transits are concerned, he did take part together with Johann Albrecht Euler and the Jesuit Fathers Christian Mayer and Gottfried Stahl in the observations of the transit of 3–4 June 1769 in the observatory of the Petersburg

Academy of Sciences (Moutchnik 2006)[1], but as a newcomer in practical astronomy he put no great weight on the accuracy of his own observations, nor expected others to do so. His engagement in the Venus transit project of 1769 was more theoretical and mathematical than practical and observational. Lexell's mission was nothing less than to determine, with the highest possible degree of accuracy, the solar parallax on the basis of all observations assembled world-wide in the year 1769.

2. Lexell's role in the Venus transit project

Lexell arrived in Saint Petersburg from Åbo, Finland, in the end of October 1768. The Secretary of the Imperial Academy of Sciences, Leonhard Euler's oldest son Johann Albrecht, played a key role in integrating Lexell in the scientific life of the Academy. As an Editor of the Academy's official organ, *Novi Commentarii Academiae Scientiarum Imperialis Petropolitanae*, J. A. Euler provided room to a number of lengthy and theoretically ambitious works written by Lexell in the fifteen years to come. Christian Mayer, who was engaged in Saint Petersburg from May 1769 to June 1770, taught Lexell the art of observing using astronomical instruments (Moutchnik 2006). The Secretary of the Royal Academy of Sciences of Stockholm, Pehr Wilhelm Wargentin, was another unfailing supporter and confidant of Lexell, as is evident from their largely preserved correspondence as well as from the number of Lexell's articles published in the Transactions of the Royal Academy of Sciences of Stockholm, *Kongliga Vetenskapsacademiens Handlingar*, even before Lexell was a member (in 1773).[2]

The Petersburg Academy of Sciences invested considerable resources and prestige in the Venus transit project of 1769 (cf. e.g., Bucher's contribution to these Proceedings). The individual observation reports from the various Russian-sponsored expeditions were churned out from the press and distributed across Europe with aplomb as soon as the Academy received them. Furthermore, all reports were edited in Latin in the *Novi Commentarii* as well as in a separate volume entitled *Collectio omnium observationum quae occasione transitus Veneris per Solem a. MDCCLXIX. iussu Augustae per Imperium Russicum institutae fuerunt una cum theoria indeque deductis conclusionibus* (1770)[3]. For these official reports, as well as for several subsequent publications, Lexell appears to have made the lion's share of the calculations pertaining to the solar parallax. In a letter dated 18 August 1769 (Centre for history of science at the Royal Swedish Academy of Sciences, Stockholm), Lexell reports to Wargentin that all the observation journals made in Russia of the transit of Venus had already reached the Academy at that time, except the one from Iakutsk, and that five of the respective reports had already been printed, namely those from Saint Petersburg, Kola (Stepan Rumovski), Ponoi (Jacques-André Mallet: Candaux et al.

[1] Independent observations were made at the Academy Observatory in Saint Petersburg by Christian Mayer SJ, his assistant Gottfried Stahl SJ, Johann Albrecht Euler, and Lexell. Both the beginning and the end of the transit could theoretically be observed in Saint Petersburg (where the Sun set just after ingress), but only the two lasts contacts were actually perceived distinctively (after sunrise the next day). For details, see (*Collectio omnium observationum quae occasione transitus Veneris per Solem,* 1769).

[2] For a comprehensive list of Lexell's works, see Johan C.-E. Stén's biography of Lexell (forthcoming on Birkhäuser Verlag).

[3] "Collection of all the observations that were made in the Russian Empire upon orders from Her Majesty on the occasion of the Venus transit in front of the disk of the Sun in the year 1769, with a theory and conclusions resulting thereof".

2005), Umba (Jean-Louis Pictet: Candaux et al. 2005) and Orenburg (Wolfgang Ludwig Krafft). One year after the transit of Venus, Lexell wrote to Planman about his work with the method of determining the parallax in a letter dated 25 June 1770 (Helsinki University Library Ms. Coll. 171):

> I have been commissioned to calculate [Georg Moritz] Lowitz' and [Christopher] Euler's observations [from Gur'ev and Orsk, respectively]; thus the biggest task [in the Astronomical part of the *Novi Commentarii*] will be my duty. As to the latter work I wish to wait for the observations from California and South Pacific. The rest that I can accomplish will be for my friends, since I have not yet planned to publish my calculations of the eclipse or those I plan to undertake on Venus.[4]

The first results of the Russian enterprise for the transit of Venus were published in 1770 in the second part of the fourteenth volume of the *Novi Commentarii* as well as in the separate book (*Collectio,* 1770). The individual reports from the various stations were followed by a long section entitled *Expositio methodorum, cum pro deteminanda parallaxi Solis ex observato transitu Veneris per Solem, tum pro invendiendis longitudinibus locorum super Terra, ex observationibus eclipsium solis, una cum calculis et conclusionibus inde deductis*[5] (*Collectio,* 1770, pp. 342–574). All calculations were exposited in full detail.

The anonymous author who based his calculations on the observations made in different parts of Russia as well as Greenwich, Cajaneborg (Kajaani) in Finland, Vardøhus in Norway, Prince of Wales Fort in present day Canada and Santo Domingo in the Caribbean, was able to deduce a mean horizontal parallax of approximately $8''.80$.[6] In an *additamentum* apparently inserted just before publication, further observations were included from North America – among these Chappe d'Auteroche's from Baja California. The anonymous author now switched to $8''.75$ as the most probable parallax[7]. The earliest contributions on the solar parallax from Russia were thus anonymous, perhaps to emphasize that the results were obtained as a joint venture and that nobody should take the blame if the result turned out erroneous, or get the credit if the contrary should be the case. Elsewhere in the *Collectio* (especially p. 575), however, Euler is singled out as the inventor of the method. Who

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[4] "Det är mig updragit at beräkna Lowitz och Eulers observationer, således faller det drygaste arbetet för den Class på min lott. Jag wil och hwad det förra arbetet angår afbida observationerne ifrån Californien och Zudsee. Hwad jag sedermera kan praestera blir för mina wänner, ty ännu har jag ei tänkt på at publicera hwarken mina beräkningar öfver Förmörkelsen eller de jag tilämnar öfver Venus".

[5] "Presentation of the methods used, both for the determination of the solar parallax on the basis of the observation of a transit of Venus in front of the Sun, and for finding the longitudes of places on the surface of the Earth on the basis of observations of solar eclipses, along with the calculations and the conclusions drawn upon these".

[6] *Novi Commentarii Academiae Scientiarum Imperialis Petropolitanae* Tomus XIV, Pars II, 1769 [published in 1770], pp. 518–519; repeated in *Collectio*, 1770, pp. 538–539: Parall[ax]is Solis nobis erit $\pi = 8,67$ quae respondeat distantia Solis a terra, quae hoc tempore erat 1,0154. Pro distantia media, quae unitate exprimi solet, haec parallaxis aliquanto fiet maior scilicet **8,80** quae quum referatur ad semiaxem telluris, distantia media inter centra Solis et terrae censenda erit aequalis 23436 semiaxibus terrae, hincque pro perigeo parallaxis $= 8,95$ et pro Apogeo $8,65$. (our emphasis)

[7] See *Collectio*, 1770, p. 556: Elementa autem Astronomica hinc sequenti ratione determinabuntur. [...] Parallaxis Solis Horizontalis 8,62. (Note that the quotation refers to the parallax on the day of the transit, which corresponded to a mean horizontal parallax of **8".75**.)

actually did the calculations is not stated explicitly, but according to the minutes of the academic conference, on 13 December 1770 (old style; i.e. on 24 December), Euler held a panegyric speech with an exuberant commendation for Lexell, stating in particular that (translated from the German) (*Протоколы засѣданий конференции Императорской Академии Наукъ*, 1897–1911, pp. 792–793)

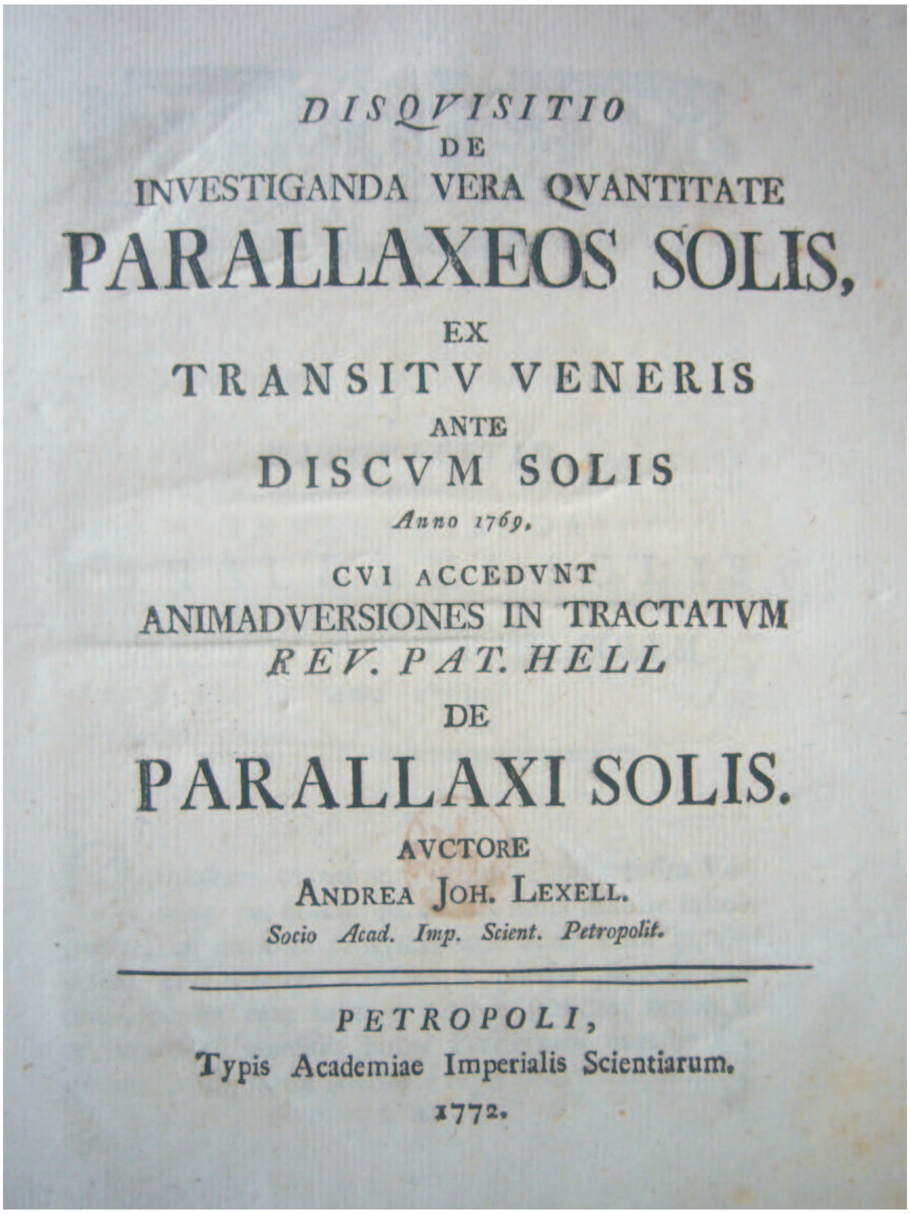
> [T]he world owes it solely to the untiring zeal of Adjoint Lexell, that the enormous expenses that were invested for the latest transit of Venus did not end up utterly wasted, in the same manner as those spent on the previous transit of 1761; without him, perhaps no one would have been able to determine from the observations made of the last transit of Venus the true parallax of the Sun, since the methods of this calculation known hitherto are entirely inadequate, as was also learned from the previous experience, and since to this day not a single scholar has proven himself to be so brilliant as to deduce only one, certain conclusion from all the observations.

Thus, starting out as an anonymous "ghost writer",[8] Lexell was gradually accorded the task of calculating the solar parallax on the Academy's behalf and soon appeared on the title pages as the sole author – no small responsibility on the shoulders of an inexperienced adjunct!

By the time the last key data sets finally reached Europe – namely the results of Captain James Cook's observation from Tahiti – three major "parallax agitators" had established themselves on the international stage. The three were Maximilian Hell, who used his own observation from Vardø along with the non-European observations to argue for a solar parallax of $8''.70 \pm 0''.01$; Anders Planman in Åbo, who used his own observation from Kajaani along with the non-European observations to argue for $8''.50$ or even lower; and Jérôme de Lalande in Paris, who essentially supported Planman and discarded Hell's observations. Lexell's own deductions were published in the Swedish *Handlingar* (Lexell 1771a,b), and in the Russian *Novi Commentarii* (Lexell 1771c). In the two latter publications, having taken the results of the observations made in Tahiti into account, Lexell settled for the parallax of 8.68 arcseconds.[9] After several attempted calculations Lalande landed on $8''.50$, or $8''.60$ as a maximum. Honour and prestige was at risk, and Lexell soon found himself bombarded by letters from Planman, Lalande, and Hell. His name was also mentioned in the columns of the major journals and magazines of learning in France and Germany. Whilst Lalande and Hell attacked each other, Lexell wrote a long treatise that was published as a monograph in Saint Petersburg late in the year 1772 (Lexell 1772b, see Fig. 1): *Disquisitio de investiganda vera quantitate Parallaxeos Solis, ex Transitu Veneris ante discum Solis Anno 1769, cui accedunt animadversiones in tractatum Rev. Pat. Hell de Parallaxi Solis* (Deliberation on the investigation of the true quantity of the solar parallax based on the transit of Venus in front of the disk of the Sun in the year 1769, to which is added comments on Honourable Father Hell's treatise *de Parallaxi Solis*). Instead of discarding Vardø in

[8] Thus, the minutes of the conferences of the Petersburg Academy informs that Lexell presented the reports of Georg Moritz Lowitz, Christopher Euler and Petr Inochods'ev during conference sessions at the Academy (cf. (*Протоколы засѣданий конференции Императорской Академии Наукъ*, 1897–1911) for 5 July 1770; 13 August 1770; and 28 February 1771) and that he edited at least the report of Lowitz for printing in the *Novi Commentarii*.

[9] The reason why Lexell's posterior values differ from the earlier and more correct value obtained under Euler's direction, might be in the rather arbitrary importance he gives to the observations of the inner contacts, which tends to make the parallax value a bit too small (Verdun 2010).



DISQVISITIO
DE
INVESTIGANDA VERA QVANTITATE
PARALLAXEOS SOLIS,
EX
TRANSITV VENERIS
ANTE
DISCVM SOLIS
Anno 1769,
CVI ACCEDVNT
ANIMADVERSIONES IN TRACTATVM
REV. PAT. HELL
DE
PARALLAXI SOLIS.
AVCTORE
ANDREA JOH. LEXELL.
Socio Acad. Imp. Scient. Petropolit.

PETROPOLI,
Typis Academiae Imperialis Scientiarum.
1772.

Figure 1. Front page of Lexell's great work (Lexell 1772b) on the solar parallax. Photo: Johan Stén. Collection of Mr Ilkka Paatero.

favor of Kajaani or *vice versa*, Lexell used a mean value of the two, arguing that there was no way to say which of the two observations that contained an error. Lexell's verdict was now $8''.63 \pm 0''.06$.

The debate on the solar parallax continued until 1775 with several printed contributions from the pen of Lexell (Lexell 1771a,b,c; 1772a,b and 1773) as well as numerous letters to *inter alios* Anders Planman (Helsinki University Library, Ms. Coll. 171), Pehr Wargentin (Centre for history of science at the Royal Swedish Academy of Sciences, Stockholm) and Johann III Bernoulli (Universitätsbibliothek Basel, Mscr. L I a 703). Although he was generally more sober than Hell, Lalande and Planman in his printed publications, Lexell ventilated his personal opinion on the other antagonists in the controversy in his earnest and outspoken letters to Wargentin[10]. For example, in a letter dated 3 April 1773, Lexell admits to Wargentin (Centre for history of science at the Royal Swedish Academy of Sciences, Stockholm):

> The only thing I have wanted to prove is that Father Hell has erred when he has criticized the calculations of the XIVth volume of the *Novi Commentarii* and that his own calculations are so severely erroneous, that nothing can be concluded from them. I am delighted to have been able to show Father Hell some reason; as to Planman I am more in despair, although he admitted to me when I visited Åbo that there is no reason to believe Father Hell's observation to be invented.[11]

To Lexell, the use of transparent calculations in a cool-headed, disinterested quest for Truth was all-important. His agenda was not to prove some observer's outstanding qualities or to question the credibility of any observer in particular. By operating with mean values between several observations he sought to find a statistically credible value. He had no illusions of being able to fix the solar parallax at a tiny fragment of an arcsecond; rather, he was careful to state the limits of doubt in all conclusions. Even more important than the result itself was the explanation of exactly how he had arrived at his result. In this sense, he was quite unusual in an academic environment riddled with personal ambitions and rivalry.

However, in his private correspondence, another, more temperamental side became visible. Having shown that Hell's arguments against his calculations were groundless and that the logic of Hell's own deductions were defective (Lexell 1772b, 1773), Lexell still feared that not only his own reputation was endangered, but also that of his superior Euler. Thus, in long and detailed letters he tried to convince his friends Wargentin in Stockholm, Planman in Åbo and Johann III Bernoulli in Berlin, of the solidity of his arguments and warning them against believing those of Hell. He always respected Wargentin's wise and diplomatic response, but was rather disappointed at the positions that Planman and Bernoulli had taken. He also showed a bold directness in his letters to senior astronomers such as Professor Planman (letter dated 10 February 1774, Helsinki University Library, Ms. Coll. 171):

> Allow me to admit that I had hoped for a little more consideration from you in this matter, which is not the trickiest one and in truth requires more of a sound logic and critique than sophisticated mathematics. Least of all had I anticipated that you, Herr Professor, would have put up against my reasons with a certain authority, which I, for all my appreciation of your personal character and qualities, cannot bring myself to approve. You may be convinced that not even Euler, the great Euler, is capable of convincing me on his mere authority, no more than anybody else.[12]

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[10] For quotations, see Aspaas (2010) or Aspaas (2012, pp. 322–326).

[11] "Det enda som jag welat bewisa, är at Pat: Hell haft orätt då han criticerat de räkningar som förekomma uti XIV Tomen af Comment: samt at Hans egna räkningar äro så swårt felaktiga, at af dem ingen ting kan slutas. Det fägnar mig, at jag kunnat bringa Pat: Hell til så mycket billighet, om Planman miströtar jag mera, likwäl måste han medge mig då jag war i Åbo, at ingen anledning är, at misstänka Pat: Hell observation för at wara updiktad".

[12] "Emedlertid må Herr Professorn tillåta mig at upriktigt tilstå, det jag hade förmodat lite mera öfwerläggning af Herr Professorn hwad detta ämne angår, som wäl ei är af de aldra benigaste

Even when the heat of battle had cooled down, Lexell writes in a serious and ironic tone to Bernoulli, reproaching him for being unconcerned and ignorant (24 December 1775):

> Enfin j'ai reçu le supplément de l'Abbé Hell sur la parallaxe[13], je l'ai trouvé tel que je me l'avois imaginé et même pire encore. Il faut bien, que vous Monsieur, l'aviez parcouru bien à la hâte, lorsque vous m'écrivîtes il y a un an, que j'aurai raison d'être bien content de l'Abbé Hell. J'en conviens volontiers, si je pourrois m'imaginer que ce soit par complaisance pour moi, qu'il persiste encore sur les objections, qu'il a faites contre les calculs sur la parallaxe dans le XIV Tome des Commentaires; qu'il defend toutes les fautes qu'il avoit commis lui-même; qu'il fait imprimer une de mes lettres[14] sans m'en demander la permission; qu'il y ajoute quantité des notes en partie triviales et pour la pluspart absurdes; qu'il s'approprie le droit de corriger ou plustôt pervertir mes calculs sans les entendre; qu'il propose plusieurs insinuations et imputations odieuses contre moi. Je dis, que si je serois assez bête pour me persuader, que tout ceci soit à mon avantage, j'aurais beaucoup à me louer de l'Abbé Hell. Soyez vous-même Monsieur, mon juge s'il vous plaît. Mais permettez aussi que je remarque le contraste singulier, qu'il y a entre votre conduite envers l'Abbé Hell et moi. Vous approuvez la conduite de l'Abbé Hell, sans l'avoir examiné et quand je vous demande votre sentiment sur des choses controversées entre lui et moi, vous, vous excusez par votre peu de temps. Je ne vous ai demandé, que vous disiez quelque chose au désavantage du caractère personnel de l'Abbé Hell, j'ai seulement voulu sçavoir si selon votre sentiment il avoit tort sur une telle question, ou non?

After this temperamental outburst the matter would no longer be discussed, and the correspondence continues in a respectful and polite tone on other subjects.

3. The methods of Euler and Lexell to determine the solar parallax

The methods to determine the solar parallax in the eighteenth century typically involved the following steps (Verdun 2004, 2010): 1) the measured contact moments were corrected from errors due to clock drift, giving the primary observables; 2) the epoch or the exact duration of transit were obtained as the secondary observables; 3) these observables were next reduced to the Greenwich or Paris meridian or to the Earth's center and subsequently compared to theoretical values obtained for the same location; 4) by averaging the differences between each pair of reduced observables (the observed and theoretically estimated ones) a set of averaged observed

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och i sanning mera fordrar en sund logica och riktig critique, än diupsinnig Mathematique. Aldra minst hade jag wäntat, at Herr Professorn emot mina skäl, allena tyckes wilja sätta en wiss auctoritet, som med al aktning för Herr Professorns person och egenskaper, jag icke kan finna mig uti at erkänna. Herr Professorn kan wara öfwertygad at Euler, den stora Euler, ingenting förmår öfwer mig blott genom auctoritet, mycket mindre någon annan".

[13] M. Hell: *Supplementum ad Ephemerides Astronomicas Anni 1774 ad Meridianum Vindobonensem*, Vienna, 1773.

[14] cf. (Lexell 1773). Lexell himself was contrary to publishing the letter. It is of course supplemented with Hell's own footnotes and refutations of Lexell's arguments.

differences $\Delta_{\rm obs}$ and theoretical differences $\Delta_{\rm theory}$ were obtained; 5) for each observation pair, the observed parallax was then given by the product of $\Delta_{\rm obs}/\Delta_{\rm theory}$ and the theoretical *a priori* parallax estimate $\pi_{\rm theory}$. All the parallax values obtained in such a way were subsequently averaged and scaled according to the mean distance between the Sun and the Earth. Of course, for this procedure to be valid, the modeling equation has to be linear, which was only assumed, however.

Several variations of the method of averaging and comparison were used in 1769. As can be expected, they did not give uniform results, which caused much confusion and disagreements among the scientists. The solar parallax being an absolute constant, every pair of observation of Venus on the Sun's disk should obviously produce the same value for the parallax π. The fact that the results nevertheless differed from each other was mainly due to measurement errors, including imprecise instruments, individual reading errors, the effect of atmospheric refraction and so on. This posed a new problem of modeling: how to describe, by means of physical laws, the true observables, including the different sources of error. The problem was rarely understood at the time; among the few who did were Euler (and Lexell), as well as the French astronomer and mathematician Achille Pierre Dionis du Séjour (Verdun 2004).

Euler's method was presented as a section of the large second part of the fourteenth volume of the *Novi Commentarii*, pp. 321–554.[15] In the method, which also Lexell adopted (in its essentials) in his subsequent studies, the solar parallax was determined by fitting as many reliable measured observables to the observation equations concurrently – instead of pairwise comparison and averaging, which had been used previously, and was still used by most astronomers involved in the calculations based on the Venus transit of 1769. As indicated in the title of the work, the method was also applicable to the determination of longitude following the solar eclipse in 1769.

In Euler's method, the observables were described by mathematical relationships. All physical laws involved in the process and the quantities in these equations, the so-called model parameters, which are known only approximately because of the errors inherent in the observations, were modeled. The goal of the process is to adjust the parameters so as to minimize the sum of all estimation errors. The so-called observation equations were derived in three steps: 1° the geocentric angular distances between the center of the Sun's and Venus' discs at conjunction are determined from astronomical tables, 2° these elements are reduced to the pole of the equator and from there to the zenith of any place on Earth, and 3° the apparent distance between the centers of the Sun's and Venus' discs are determined in terms of the desired quantity, the solar parallax π. The derived observation equation contained several variables and constants to be determined. The ensuing adjustment process involved the following phases: minimizing the number of parameters by linear combinations of equations, grouping the equations according to the four contact moments, averaging the equations to determine approximations of the parameters searched for, computing more accurate theoretical elements and setting up new equations with correction terms as new unknowns. Finally, the corrections are determined so that the sum of the estimation errors is as small as possible.

Euler's method thus involved at least two novel ingredients, viz. statistical data processing and the minimization of estimation errors. The calculations were obviously very time consuming in those days – for today's personal computers they

[15] Also in *Leonhardi Euleri Opera Omnia* Ser. II, Vol. 30, pp. 153–231.

would be the work of a few seconds – but considering that the theory of measurements, linear algebra and the method of least squares had not yet been developed, the results were astonishingly accurate.

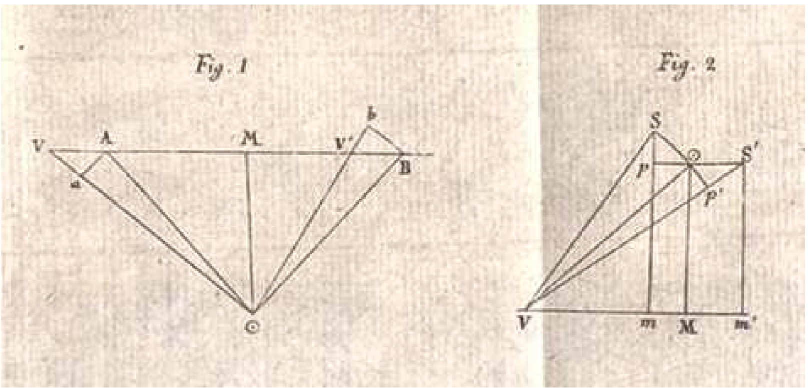

Figure 2. Pertinent to the geometry of the transit of Venus (Lexell 1771a). On the left, the effect of the parallax on the observation of Venus at ingress and egress. On the right, the corrections due to uncertainties in the observation of Venus' position.

The method Lexell used in his own contributions contains essentially the same ingredients as Euler's described above. Basically, it is given in (Lexell 1771a) written for the Swedish audience. Let us consider "Fig. 1" to the left in Fig. 2 (Lexell 1771a): Let VMV′ be Venus' orbit, ⊙ the center of the Sun, and ⊙M the smallest distance between the Sun and the orbit of Venus. Further, let A be the position of Venus at ingress (entry) and B its position at egress (exit), for either inner or outer contact, as seen from the center of the Earth. For the outer contacts, ⊙A and ⊙B equal the sum of the semi-diameters of the Sun and Venus. Correspondingly, for the inner contacts, ⊙A and ⊙B equal the difference of the semi-diameters of the Sun and Venus. Lexell's estimate for the semi-diameter of the Sun at the moment of transit was $947''$ and for Venus $29''$. Thus, for external contacts ⊙A = ⊙B = $976''$ and for internal contacts $918''$. The geocentric latitude of Venus (degrees above or below the ecliptic) was $10'13.4''$ and hence, the smallest distance ⊙M = $606.7''$. Thus, for the external contact AM = $764.52''$, the corresponding duration for AM = $3^{\mathrm{h}}11'8''$ and hence, for the inclination of the ecliptic, the angle will be A⊙M = $51°33'56''$. Similarly, for the internal contact AM = $688.94''$, the duration of AM = $2^{\mathrm{h}}52'14''$ for the elements of the Sun on June 3. Hence, the angle will be A⊙M = $48°37'55''$. Now, V and V′ are the positions of Venus when, somewhere on the Earth, a contact (external or internal) is observed at ingress or egress, respectively. If ⊙a = ⊙A and ⊙b = ⊙B, then Va and V′b represent the parallax effects in the directions V⊙ = V′⊙. In the direction of the orbit VA, the effect is obtained approximately by the relationship VA = Va $\sin$(VAa) $\csc$(VAa) when the *a priori* estimate $8.''5$ for the solar parallax is used.

Next, the influence of the uncertainty of ⊙A and ⊙M is studied (cf. "Fig. 2" to the right of Fig. 2). First, let ⊙V be constant while ⊙M suffers a slight augmentation. From V, a circular arc CS is drawn, and the line Sm parallel to ⊙M. Sm denotes the true distance between the centers of the Sun and Venus. Further, ⊙p is drawn parallel to VM, and Sp is the small correction needed for ⊙M. Then, in the triangle S⊙p, p⊙ = Mm = Sp, which is the amount VM has diminished. Second, let ⊙M be constant while $\odot$V endures an augmentation S$'p'$. If S$'m'$ is parallel to $\odot$M, then M$m' = \odot$S$' = p'$S$'$. Hence, the total correction is VM $=$ S$'p' \sec(\odot$VM$)$ $-$ S$p \tan(\odot$VM$)$. When the parallax-effect in the direction $\odot$V is denoted $\alpha\pi$, and denoting by π the horizontal parallax, and setting S$p = y$, and S$'p' = \mu$ for external contacts and ν for internal contacts, we get the equations

$$\mathsf{VM} = 764.52 \pm \alpha\pi \sin \mathsf{V}a\mathsf{A} \csc \mathsf{VA}a - y \tan \odot\mathsf{VM} + \mu \sec \odot\mathsf{VM},$$

$$\mathsf{VM} = 688.94 \pm \alpha\pi \sin \mathsf{V}a\mathsf{A} \csc \mathsf{VA}a - y \tan \odot\mathsf{VM} + \nu \sec \odot\mathsf{VM},$$

for external and internal contacts, respectively. Correspondingly, knowing the speed of Venus with respect to the Sun, the durations for the respective distances VM are

$$\mathsf{T} = 3^{\mathrm{h}}11'8'' \pm 15(\alpha\pi \sin \mathsf{V}a\mathsf{A} \csc \mathsf{VA}a - y \tan \odot\mathsf{VM} + \mu \sec \odot\mathsf{VM}),$$

$$\mathsf{T} = 2^{\mathrm{h}}52'14'' \pm 15(\alpha\pi \sin \mathsf{V}a\mathsf{A} \csc \mathsf{VA}a - y \tan \odot\mathsf{VM} + \nu \sec \odot\mathsf{VM}).$$

Finally, for each site of observation, expressions are formed for the external contact at ingress T, the internal contact at ingress T$'$, the internal contact at egress T$''$ and the external contact at egress T$'''$. Then a series of equations follow for each observation site and when they are compared to the actually measured times at respective stations, they lead to an over-determined system of equations, to which a best possible estimate is sought (with respect to some unspecified norm), allowing certain corrections in longitude (μ and ν) and latitude y.

4. Conclusions

Anders Johan Lexell had high scientific ideals. While wishing to stand aloof from petty quarrels and vanity in his dedication to "pure science", he soon found himself in the midst of a scientific controversy guided by other factors entirely than the pursuit of Truth. To Lexell's dismay, the vices of personal ambition and protonationalistic sentiments dominated the academic scene. Hell, Lalande and Planman were all senior professors; Lexell a mere adjunct. Being the youngest and least merited, his "claim to fame" on the international stage was perhaps less obvious, but acting as he was on behalf of the high-ranking Imperial Academy of Sciences of Saint Petersburg and the world-famous Euler, his calculations nevertheless gained considerable attention. The determination of the solar parallax had been Lexell's first test as an astronomer. When reading through his various publications on the subject, one may conclude that he succeeded better than most contemporaries to focus on the scientific rather than the non-scientific. Lexell was always careful to argue *ad rem* instead of *ad hominem*. Apart from his Latin and Swedish publications already mentioned, Lexell's German contributions in the *Astronomisches Jahrbuch* of Berlin (1775a,b) published in 1775 contain the summit of his contribution to parallax computations, not only the parallax of the Sun, but of any star or distant object. The clarity and mathematical perspicuity of his texts make them well worth re-visiting.

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