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Dependence Uncertainty with Applications in Finance and Insurance

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Abstract

The interactions between all elements in the universe persist, whether their influence is mighty or subtle. For instance, planets move around stars, while molecules continuously engage in random motions within the void of space. The interaction can be modeled by quantifying the elements with random variables in a probability space. That is the subject of dependence modeling. This doctoral thesis focuses on the lower and upper bounds of the expectation of a measurable function $\psi : \mathbb{R}^d \to \mathbb{R}$ of d random variables, $\mathbb{E}\psi(X_1, X_2, \ldots, X_d)$, with known marginal distributions $X_i \sim F_i$, $i = 1, 2, \ldots, d$, but unknown dependence structure. Moreover, the applications of these bounds in finance and insurance are studied.

In the second chapter, we study the impact of dependence uncertainty in the case of $\psi(X_1, X_2, \ldots, X_d) = \prod_{i=1}^d X_i$. Under some conditions on the F_i, explicit sharp bounds and expressions of the corresponding copulas that attain them are obtained. A numerical method is provided to approximate them for arbitrary marginals F_i. In the special case of $d = 3$, we introduce a notion of "standardized rank coskewness," which is invariant under strictly increasing transformations and takes values in $[-1, 1]$.

Using the analytic result in the first chapter, we next show that one needs to be careful when making statements on potential links between correlation and coskewness. Specifically, we first show that, on the one hand, it is possible to observe any possible values of coskewness among normal random variables but zero pairwise correlations of these variables. On the other hand, it is also possible to have zero coskewness and any level of correlation. Second, we generalize this result to the case of arbitrary marginal distributions by sharing the absence of a link between rank correlation and rank coskewness.

In the context of finding risk bounds when $\psi(X_1, X_2, \ldots, X_d) := \mathbb{1}_{f(X_1, X_2, \ldots, X_d) > x}$, $x \in \mathbb{R}$ and the function f is supermodular, Puccetti and Rüschendorf (2012) introduce the rearrangement algorithm (RA) as a tool for (optimally) rearranging matrices. The RA also has applications in finance and operations research. The block rearrangement algorithm (BRA) proposed by Bernard and McLeish (2016) and Bernard et al. (2017) is, from a theoretical

perspective, more accurate than the RA but also slower. In the fourth chapter, we aim to improve the convergence speed and accuracy of the BRA. That is, we seek to find the optimal sequence of block (submatrix) sizes rather than simply picking the size of the blocks in a fully equiprobable way, as proposed in Bernard and McLeish (2016) (standard BRA). To do so, we refine the BRA by sampling the block size from a Beta distribution with two parameters that can evolve over the different steps. We make recommendations for the best choice of these two parameters. Specifically, our proposed BRA Beta is designed to choose large block sizes at the beginning (like standard BRA) and then change to small sizes (like RA). A numerical study shows that BRA Beta outperforms the other variants of the rearrangement algorithm in the literature.

Recent literature has identified the two-dimensional coskewness as one of the most important factors of portfolio optimization. The fifth chapter is dedicated to studying the lower and upper bounds on $\mathbb{E}\psi(X_1, X_2, \ldots, X_d)$ in the case of $\psi(X_1, X_2, \ldots, X_d) = X_1 X_2^2$. In particular, we provide the risk bounds for arbitrary marginals and explicit dependence structures for some conditions, such as the symmetric distribution with zero mean. Moreover, an algorithm is designed to find dependence structures for arbitrary marginals. We then apply the analytic and numerical results to find the bounds on the two-dimensional coskewness and discover some related properties.