VRIJE
UNIVERSITEIT
BRUSSEL

Thesis submitted in fulfilment of the requirements for the award of the degree of Doctor of Business Economics (Doctor in de Toegepaste Economische Wetenschappen)

Essays in Quantitative Risk Management and Financial Engineering

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Abstract

This PhD thesis is a collection of three essays that embraces different topics, namely option-implied risk measures, optimal portfolio choice problems, and continuous-time autoregressive moving-average processes, in which the common thread is the use of numerical procedures in order to find an answer to a financial query or a more suitable alternative to existing methodologies.

As a matter of fact numerical techniques play a fundamental role and an attempt of offering an answer to mathematical problems and applications for which an exact solution is prohibitively demanding or impossible to retrieve. It is clear that, in contrast to a symbolic (analytical) solution, the numerical result provides just an approximation of the answer. The use of numerical approaches is not just relegated to the fields, for instance, of physics and engineering, but it is largely employed also in quantitative finance and actuarial sciences. The reason of a such wide success relies on the fact that numerical techniques are implemented, not only in the academia, but also in the industry for solving real life problems.

Indeed, in this PhD thesis, we design, implement and analyse some numerical procedures: i) an "exact" simulation method which can be used for simulating the underlying terminal asset value for any model in which closed-form or semi-closed-form expressions for the option prices are known; ii) an approach for solving the multivariate optimal portfolio selection problem; and iii) an approximation scheme for the transition density of a continuous-time autoregressive moving-average (CARMA) process driven by a time-changed Brownian motion.

A brief summary of the chapters of the doctoral thesis is presented. In the second chapter, we propose a novel model-free approach for extracting the risk-neutral quantile function of an asset using options written on this asset. The approach has a two-fold application. First, from the expression of a quantile as the minimizer to an optimization problem over a weighted sum of prices of call and put options, the developed approach is able to compute, bypassing numerical inversion, the risk-

neutral quantiles and thus implied VaR and implied TVaR. Second, it is shown how for a given stochastic asset model the approach makes it possible to simulate the underlying terminal asset value under the risk-neutral probability measure directly from option prices.

In the third chapter, we develop two numerical procedures for solving multivariate optimal portfolio choice problems. When the risk sharing problem (in the absence of a financial market) can be solved explicitly, the multivariate optimal portfolio choice problem reduces to a one-dimensional problem using the quantile approach and the numerical procedure, named Multivariate-to-Univariate, can be used. When the dimensionality of the problem cannot be reduced, then the multivariate optimal portfolio choice can still be solved using a slower numerical procedure named General Numerical approach.

In the fourth chapter, we focus on a CARMA model driven by a time-changed Brownian motion (TCBm-CARMA) and we present an approximation scheme for the transition density, constructed based on the Gauss-Laguerre quadrature. The approximated transition density is a finite mixture of normals and an estimation method that maximizes the likelihood function is introduced. Furthermore, when the underlying asset follows a TCBm-CARMA process, formulas for the futures term structure and for option prices written on futures are provided.