SCIENCES & BIOENGINEERING SCIENCES

The Research Group
Mathematics and Data Science

has the honor to invite you to the public defence of the PhD thesis of

Jonathan Mannaert

to obtain the degree of Doctor of Sciences

Title of the PhD thesis:

Boolean functions with a geometric domain: a place where algebra and combinatorics meet

Supervisor:
Prof. dr. Jan De Beule (VUB)

The defence will take place on

Thursday, June 12, 2025 at 4 p.m.

VUB Etterbeek campus, Pleinlaan 2, Elsene, auditorium I.0.01

Members of the jury

Prof. dr. Ann Dooms (VUB, chair)
Dr. Sam Mattheus (VUB, secretary)
Prof. dr. Philippe Cara (VUB)
Prof. dr. Marie-Anne Guerry (VUB)
Prof. dr. Leo Storme (UGent)
Prof. dr. Gianira Alfarano (Université de Rennes, FR)
Prof. dr. Ferdinando Zullo (Università degli Studi della Campania « Luigi Vanvitelli », IT)

Curriculum vitae

Jonathan Mannaert obtained his master's degree in mathematics at Ghent University in 2019.

Afterwards, he started as a teaching assistant in the Department of Mathematics & Data Science of the Faculty of Science and Bioengineering sciences of the Vrije Unviersiteit Brussel.

His doctoral research resulted in 10 publications in peer-reviewed journals. In addition, during his PhD, Jonathan has been teaching several exercise classes, supervising bachelor and master students, and he was also involved in various outreach events to promote mathematics.

Abstract of the PhD research

In 1982 P. Cameron and R. Liebler investigated collineation groups of the finite three dimensional projective space PG(3,q) having equally many orbits on the lines as on the points. It turned out that the line orbits of such collineation groups satisfy particular algebraic and geometric properties. These line classes were initially called Cameron-Liebler line classes in PG(3,q). It was long thought that a classification of these objects could be obtained eventually, and it was conjectured that only trivial line classes could exist. A major breakthrough in the investigation arose in 1999 when A. Bruen and K. Drudge disproved this geometrical conjecture, revitalizing the topic. A second important year was 2019, when a generalization of these line classes to sets of k-spaces in PG(n,q) were described. Furthermore, also in 2019, these generalizations, now called Cameron-Liebler sets, where studied for the first time as Boolean functions of degree 1 on a geometric domain. This viewpoint gave new possibilities to further investigate them.

In this thesis we will carefully explore **Boolean functions derived from geometric objects.** Initially, we will focus on the original problem and restrict to degree 1. This includes the extension and strengthening of known existence conditions for Cameron-Liebler sets of k-spaces by using the underlying algebra. These algebraic techniques also provide a deeper understanding and have shown to be extremely successful. They also provide a sensible way of extending these concepts to affine geometries. Secondly, we extend our investigation to Boolean functions of general degree over a finite projective space PG(n,q). Many existing results for degree 1 functions seem naturally generalizable, but this turns out not to be straightforward. More specifically, by mapping out a deep connection between degree t functions and t-designs, we are able to provide strong existence conditions for these functions. Besides, we will also focus on finding interesting examples. Special attention will be given to degree 2, as it is the logical starting point.

Finally, we will translate these valuable algebraic techniques to other geometrical problems. One such example is given by m-ovoids in polar spaces, where we are not only able to extend but also strengthen existing bounds on m.

This, in addition with the traditional Cameron-Liebler problems, provide **a place where algebra and combinatorics meet**.