CATEGORICAL CONSTRUCTIONS, BRAIDINGS ON MONOIDAL CATEGORIES AND BICROSSED PRODUCTS OF HOPF ALGEBRAS

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ABSTRACT

The thesis contains three chapters: I. *Categorical constructions for Hopf algebras and related topics*, II. *Unified products*, III. *Classifying bicrossed products of quantum groups. Deformations of a Hopf algebra and descent type theory.*

The first chapter proves that the forgetful functor from the category of Hopf algebras to the category of (bi)algebras has a right adjoint and the limits in the category of Hopf algebras are described. Moreover, the braidings on the category of bimodules over an algebra A are investigated and several equivalent descriptions for the center of the category of bimodules over A are provided.

In the second chapter the unified product is introduced for the categories of groups and Hopf algebras as an answer to the extending structures problem. Equivalent descriptions for the unified product in the category of Hopf algebras are provided and a classification theory is presented. Some properties of the unified product (such as the coquasitriangular structures) are investigated. Special cases of the unified product such as the crossed product of Hopf algebras and the bicrossed product of Hopf algebras are also studied.

The last chapter deals with the bicrossed product of Hopf algebras. The main theme of this chapter is the so-called *bicrossed descent theory* which asks for the description and classification of all factorization A-forms of an extension E of A. A factorization A-form of E is a Hopf algebra H such that E factorizes through A and H. Let $(A, H, \triangleright, \triangleleft)$ be a matched pair of Hopf algebras. The Hopf algebra H is deformed to a new Hopf algebra H_r, using a certain type of unitary cocentral map $r : H \to A$ called a descent map of the matched pair $(A, H, \triangleright, \triangleleft)$. This is a general deformation of a given Hopf algebra and it is of interest in its own right. Let H be a given factorization A-form of E. The description of forms proves that $\mathbb{H}$ is a factorization A-form of E if and only if $\mathbb{H}$ is isomorphic to H_r, for some descent map $r : H \to A$. The classification of forms shows that there exists a bijection between the set of types of isomorphisms of all factorization A-forms of E and a cohomological type object $\mathcal{HA}^2(H, A \,|\, (\triangleright, \triangleleft))$. In particular, the factorization index of E/A is computed by the formula $[E : A]^f = |\mathcal{HA}^2(H, A \,|\, (\triangleright, \triangleleft))|$. Several examples are provided and the bicrossed descent theory for groups is derived.