THE KADISON-SINGER PROBLEM AND THE BOURGAIN-TZAFRIRI RESTRICTED INVERTIBILITY THEOREM

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Abstract

One of the numerous equivalent formulations of the Kadison-Singer problem is asking whether every $n \times n$ matrix A with zeros on the diagonal admits a block decomposition in which every diagonal block has small norm and the number of needed blocks is independent of n. In 1987, in their celebrated restricted invertibility theorem, Bourgain and Tzafriri proved that A admits at least one large diagonal block of small norm. In 2013, the Kadison-Singer problem was entirely solved by Marcus, Spielman and Srivastava, using totally unexpected methods. This in turn led to optimal estimates for the restricted invertibility theorem. I will explain these results, as well as a much more general conjecture for maximal abelian subalgebras of arbitrary von Neumann algebras, as I proposed in a joint work with Sorin Popa.