In the study of symmetries in mathematics, groups, i.e. sets with a binary operation satisfying some specific rules, play an important role. Many questions pop up when one tries to represent these groups as something a computer may calculate with: matrices over rings. This is done in the so-called representation theory of groups. One such question is whether these reductions to matrices allows us to reconstruct the group. In other words, can we have two different groups which have exactly the same representations as matrices over some ring? In general, the answer, counterintuitively, appears to be yes. However, the study of what classes of groups one can distinguish is still very much open. One way to tackle this problem is to study the unit group $U(ZG)$ of an integral group ring $ZG$ over a group $G$ and determine how rigidly $G$ lies in this structure.

In the first part of this thesis, we provide a way of finding many “copies” of a subgroup $H$ of $G$ inside of $U(ZG)$, via a relatively new construction called the Bovdi units. We study the structural properties that several of such copies together can possess. We prove that when $H$ is cyclic we often can construct two of these copies such that there is almost no interplay between them.

A second main part of the thesis is trying to understand this rigidity when considering $U(ZG)$ as the symmetries of a tree. In this mathematical context, a tree is a graph without loops. It appears that some structural information can be gained on $U(ZG)$ if it has symmetries on a tree such that no point is fixed by all the symmetries at once. The converse of this property is called (FA), after “points Fixes sur les Arbres”. We prove a result which reduces the study of a small adaptation of (FA) (called (HFA)) to smaller components of $U(ZG)$.

The study of (HFA) for these smaller groups, which are linear groups over orders, form the bulk of thesis. We focus on the lower rank case since these form the biggest gap in the state-of-the-art. We deduce exactly when $U(ZG)$ has (HFA) and find a list of 10 minimal counter examples.