Abstract of the PhD research

In classical mechanics, integrable systems are characterized by a large amount of symmetry, which constrains the motion heavily and results in regular trajectories. This can be contrasted with the more irregular and ergodic motion of generic chaotic systems, usually characterized by an exponential divergence of nearby trajectories in phase space. Elevating this intuition to the quantum systems is not straightforward and new tools are required to study chaos in a quantum theory.

In recent years, black holes were discovered to be chaotic objects. These findings triggered new developments in the study of quantum chaos and resulted in the derivation of a bound constraining the propagation of small perturbations across a quantum system by a linear function of the temperature, which is saturated by black holes. Chaos seems to be ubiquitous and is now understood as an inherent feature of one of the most fundamental objects in gravity. A thorough understanding of it is therefore indispensable to discover the mysteries of gravity at the quantum level.

We start by studying a class of resonant systems that arise classically from a time-averaging procedure applied on weakly nonlinear partial differential equations, originating from both Bose-Einstein Condensate and Anti-de Sitter physics. These resonant systems are known to possess a special set of periodic solutions, amongst all the other complex trajectories that one cannot solve for. We explore the integrable structures in the quantum theory that allow for these special periodic trajectories in the semi-classical limit and discover a family of exactly solvable energy eigenstates. We furthermore use a spectral probe of quantum chaos that characterizes the statistics of neighboring energy eigenvalues to investigate whether additional symmetries are present in some of the quantum resonant systems we studied and find that all symmetries have been identified.

We then turn to investigating the properties of a dynamical probe of chaos, expected to encode the physics of the classical exponential divergence of trajectories: the commutator squared. We examine this probe in a simple Ising spin chain, where it is known to fail at identifying chaotic behavior. We show how this issue gets resolved when modifying the system by increasing the spin representation at each site, which brings the system to a semi-classical regime. Finally, we explore the chaotic features of maximally rotating black holes in 3D Anti-de Sitter spacetime, which have zero Hawking temperature. We find that the commutator squared alternates between brief periods of exponential growth and longer periods of powerlaw growth (respectively due to the right-moving nonzero temperature and left-moving zero temperature), leading to an overall cubic growth interpreted as slow scrambling.